

MODULE – 1
ANALOG ELECTRONIC CIRCUITS
OPTOELECTRONIC DEVICES

Optoelectronic Devices is the field that deals with study of devices that emit, detect and control light in the wavelength spectrum ranging from ultraviolet to far infrared. They include electrical-to-optical (convert electrical energy into light energy) and optical-to-electrical (convert light energy into electrical energy) transducers. Optocouplers also come in this broad category.

PHOTODIODES:

Photodiode is a light detector semiconductor device that converts light energy into electric current or voltage which depends upon the mode of operation.

The upper cut-off wavelength of a photodiode is given by; $\lambda_c = \frac{1240}{E_g}$

where, λ_{cc} is the cut-off wavelength in nm and E_{gg} is the bandgap energy in eV.

A normal p-n junction diode allows a small amount of electric current, under reverse bias, due to minority charge carriers. To increase the electric current under reverse bias condition, we need to generate more minority carriers. The external reverse voltage applied to the p-n junction diode will supply energy to the minority carriers, but it will not increase the population of minority charge carriers.

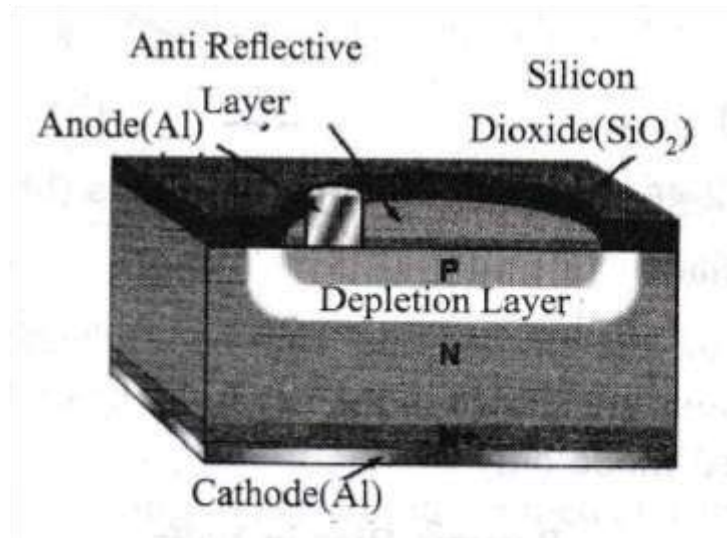
A small number of minority carriers are generated due to external reverse bias voltage. The minority carriers generated at n-side or p-side will recombine in the same material, before they cross the junction. As a result, no electric current flows due to these charge carriers. For example, the minority carriers generated in the p-type material experience a repulsive force from the external voltage and try to move towards n-side. However, before crossing the junction, the free electrons recombine with the holes within the same material. As a result, no electric current flows.

To overcome this problem, we need to apply external energy directly to the depletion region to generate more charge carriers. A special type of diode called photodiode is designed to generate more number of charge carriers in depletion region. In photodiodes, we use light or photons as the external energy to generate charge carriers in depletion region.

Construction:

The typical construction of a photodiode is illustrated in the following Figure. This example uses a construction technique called *ion implantation* where the surface of a layer of N-type is bombarded with P-type silicon ions to produce a P-type layer of about 1 μm (micrometre) thick. During the formation of the diode, excess electrons move from N-type towards P-type and excess holes move from P-type towards

N-type; this process is called *diffusion*, resulting in the removal of free charge carriers close to the PNjunction, so creating a depletion layer as shown in the following Figure.

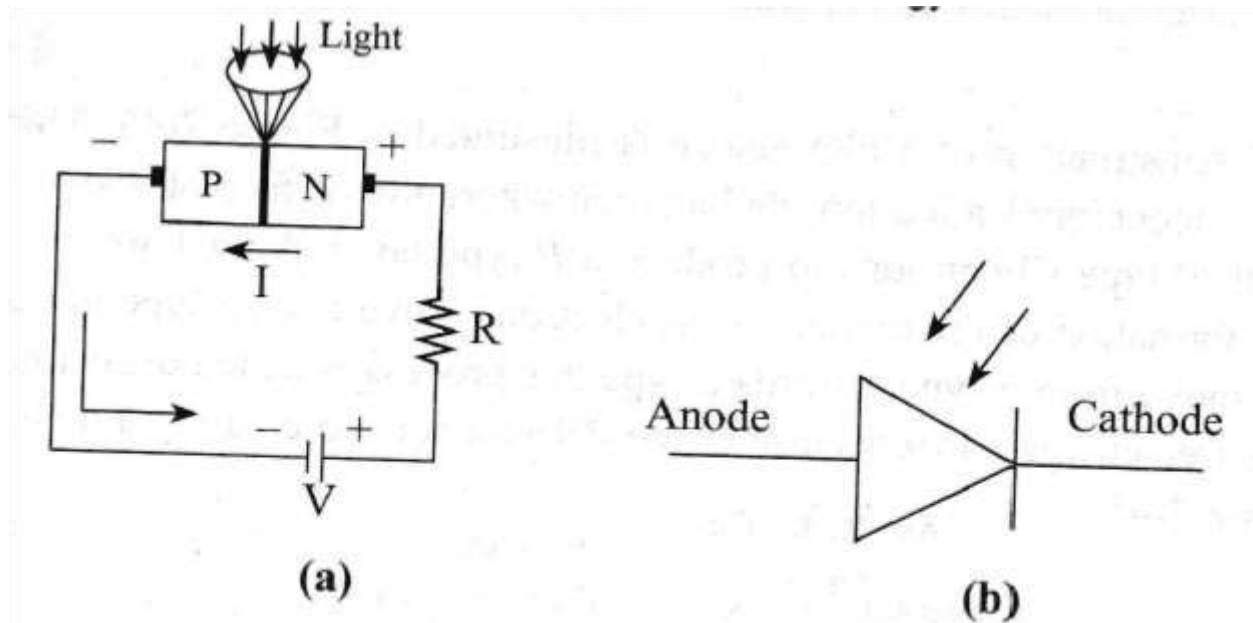


The (light facing) top of the diode is protected by a layer of Silicon Dioxide (SiO_2) in which there is a window for light to shine on the semiconductor. This window is coated with a thin anti-reflective layer of Silicon Nitride (SiN) to allow maximum absorption of light and an anode connection of aluminium (Al) is provided to the P-type layer. Beneath the N-type layer, there is a more heavily doped N^+ layer to provide a low resistance connection to the cathode.

Working Principle:

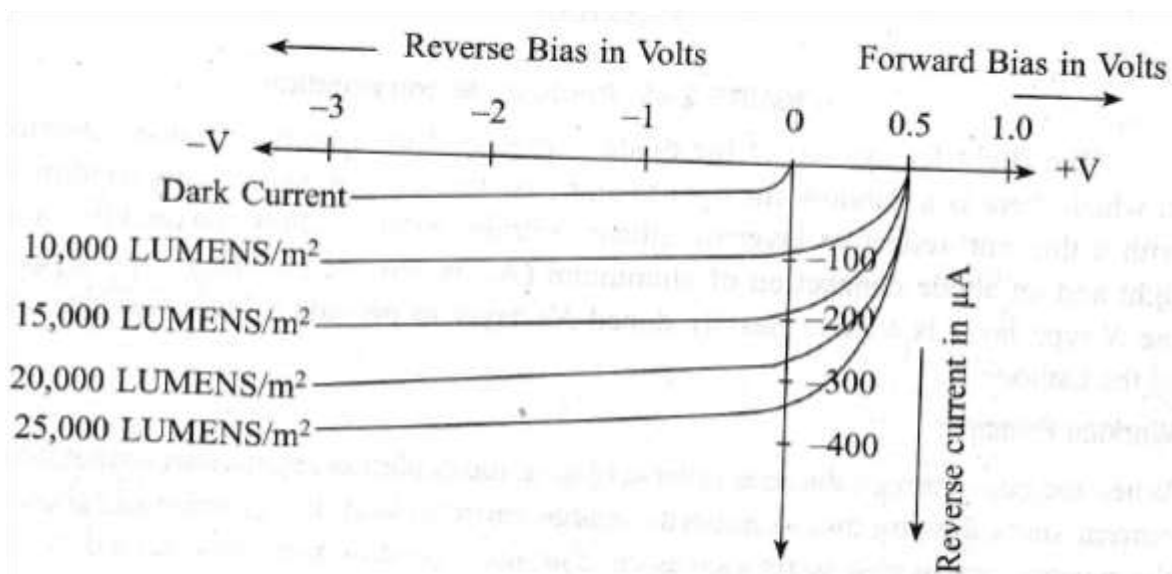
When the conventional diode is reverse biased, the depletion region starts expanding and the current starts flowing due to minority charge carriers. With the increase of reverse voltage, the reverse current also starts increasing. The same condition can be obtained in photodiode without applying reverse voltage.

The following Figure shows photo diode bias symbol. The junction of Photodiode is illuminated by the light source; the photons strike the junction surface. The photons impart their energy in the form of light to the junction. Due to which electrons from valence band get the energy to jump into the conduction band. This leaves positively charged holes in the valence band, so producing 'electron-hole pairs' in the depletion layer. Some electron-hole pairs are also produced in P and N layers, but apart from those produced in the diffusion region N layers, most will be re-absorbed within the P and N materials as heat. The electrons in the depletion layer are then swept towards the positive potential on the cathode, and the holes swept towards the negative potential on the anode, so creating a photo current. In this way, the photodiode converts light energy into electrical energy.



V-I Characteristics of Photodiode:

The characteristics curve of the photodiode can be understood with the help of the following Figure. The characteristics are shown in the negative region because the photodiode can be operated in reverse biased mode only.



The reverse saturation current in the photodiode is denoted by I_0 . It varies linearly with the intensity of photons striking the diode surface. The current under large reverse bias is the summation of reverse saturation current and short circuit current.

$$I = I_{sc} + I_0 (1 - e^{-eV/V_t})$$

Where I_{sc} is the short circuit current, V is positive for forward voltage and negative for reverse bias, V_t is volt equivalent for temperature, Δ is unity for germanium and, 2 for silicon.

Applications:

- Photodiodes are used in consumer electronics devices like smoke detectors, compact disc players, and televisions and remote controls in VCRs.

- In other consumer devices like clock radios, camera light meters, and street lights, photoconductors are more frequently used rather than photodiodes.
- Photodiodes are frequently used for exact measurement of the intensity of light in science and industry. Generally, they have an enhanced, more linear response than photoconductors.

LIGHT EMITTING DIODE (LED):

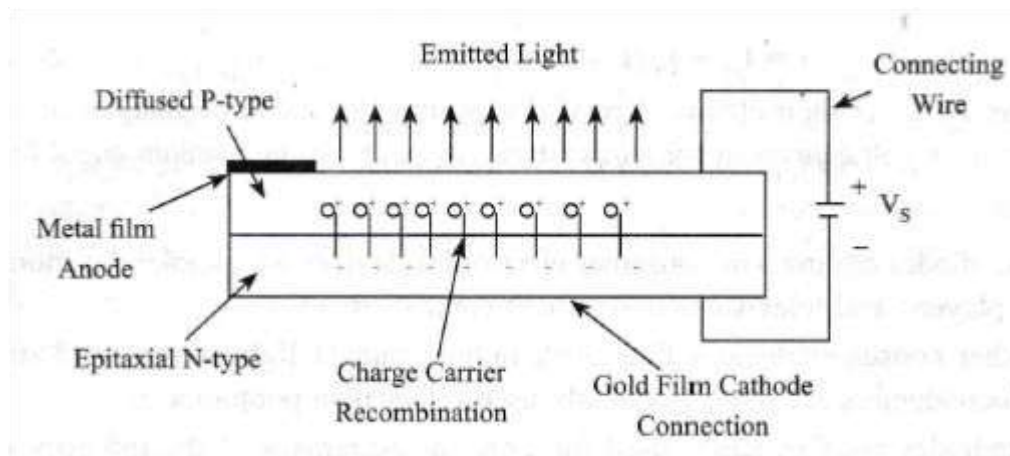
The LED is a PN-junction diode which emits light when an electric current passes through it in the forward direction. A P-N junction can convert absorbed light energy into a proportional electric current. The same process is reversed here (i.e. the P-N junction emits light when electrical energy is applied to it). This phenomenon is generally called *Electroluminescence*.

Electroluminescence is the property of the material to convert electrical energy into light energy and later it radiates this light energy. Different sizes of light emitting diodes are available in market form 1mm^2 to onward.

Construction:

The semiconductor material used in LED is *Gallium Arsenide (GaAs)*, *Gallium phosphide (GaP)* or *Gallium Arsenide Phosphide (GaAsP)*. Any of the above-mentioned compounds can be used for the construction of LED, but the color of radiated light changes with the change in material (for example, GaP material gives green/red color with forward voltage of 2.2V).

The semiconductor layer of *P-type* is placed above *N-type* because the charge carrier recombination occurs in *P-type*. Besides, it is the surface of the device, and thus, the light emitted can be easily seen on the surface. If *P-type* is placed below the *N-type*, the emitted light cannot be seen. The following Figure shows cross sectional view of diffused LED.

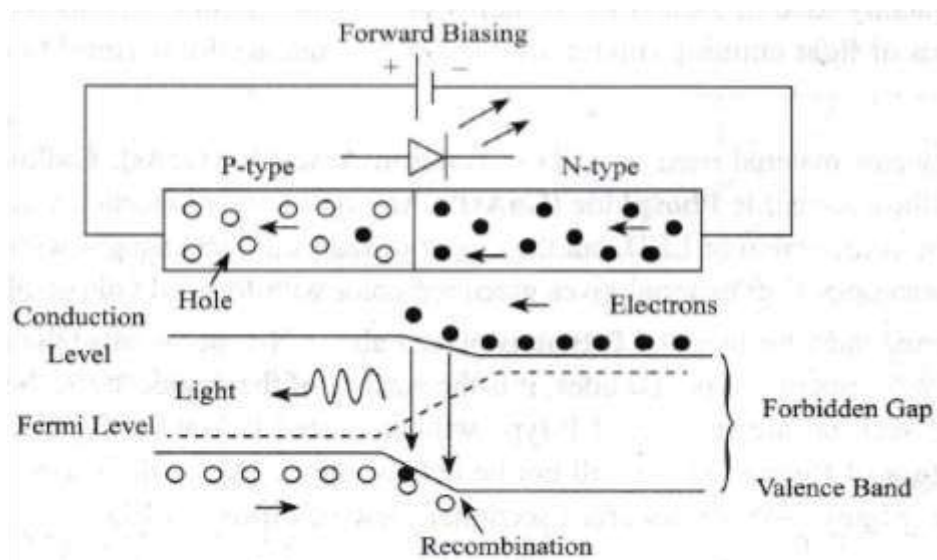


The *P-type* layer is formed from diffusion of semiconductor material. On the other side, in *N-type* region, the epitaxial layer is grown on *N-type* substrate. The metal film is used on the *P-type* layer to provide anode connection to the diode. Similarly, Gold-film layer is coated on *N-type* to provide cathode connection. The Gold-film layer on *N-type* also provides reflection from the bottom surface of the diode. If any significant part of radiated light tends to hit bottom surface then that will be reflected from the bottom surface to the device top surface. This increases LED's efficiency.

Working Principle:

The charge carriers recombine in a forward-biased P-N junction as the electrons cross from the N-region and recombine with the holes existing in the P-region. Free electrons are in the conduction band of energy levels, while holes are in the valence energy band. Thus the energy level of the holes is less than the energy levels of the electrons. Some portion of the energy must be dissipated to recombine the electrons and the holes. This energy is emitted in the form of heat and light.

The working of the LED depends on the quantum theory. The quantum theory states that, when the energy of electrons decreases from the higher level to lower level, it emits energy in the form of photons. The energy of the photons is equal to the gap between the higher and lower level, as shown in the following Figure.



The LED is forward biased, which allows the current to flow in the forward direction. The flow of current is because of the movement of electrons in the opposite direction. The recombination shows that the electrons move from the conduction band to valence band and they emit electromagnetic energy in the form of photons. The energy of photons is equal to the gap between the valence and the conduction band.

Color of light can be determined by the band gap of semiconductor material.

Applications:

- LEDs are used in remote control systems such TV or LCD remote.
- Used in traffic signals for controlling the traffic crowds in cities.
- Used in digital computers for displaying the computer data.
- Used in electronic calculators for showing the digital data.
- Used in digital watches and automotive heat lamps.

PHOTOCOUPLER:

Photocoupler or Optocoupler is a device that transfers electrical signals between two isolated circuits by using light.

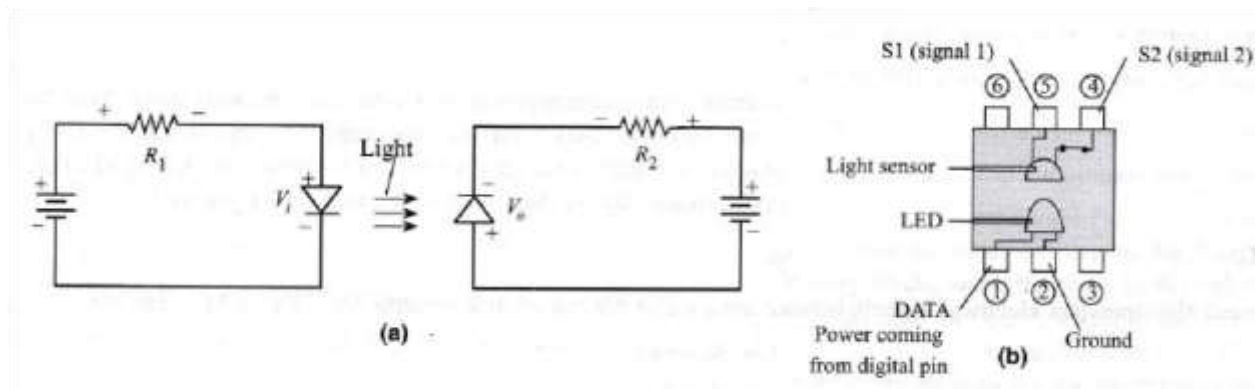
Photocouplers are used in many functions: they can be used to link data across two circuits; they can be used within optical encoders, where the optocoupler provides a means of detecting visible edge transitions on an encoder wheel to detect position, etc., and they can be used in many other circuits where optical links and transitions are needed. As a result, optical couplers or photocouplers are found in many circuits.

Construction:

All optocouplers consist of two elements: a light source (a LED) and a photosensor (a photoresistor, photodiode, phototransistor, silicon-controlled rectifier (SCR), or triac); which are separated by a dielectric (non-conducting) barrier.

Working Principle:

When input current is applied to the LED, it switches ON and emits infrared light; the photosensor then detects this light and allows current to flow through the output side of the circuit; conversely, when the LED is off, no current will flow through the photosensor. By this method, the two flowing currents are electrically isolated. It consists of LED and photodiode; where the circuits are isolated electrically. In the following Figure, LED is forward biased, photodiode is reverse biased and output exists across R_2 .



The Figure (a) describes the basic operation of an optocoupler. When current is not being applied via Pin 1, the LED is off, and the circuit connected to Pins 4 and 5 is experiencing no current flow. When power is applied to the input circuit, the LED switches on, the sensor detects the light, closes the switch and initiates current flow in the output circuit, as shown in the Figure (b).

Applications:

- Input and output switching in electronically noisy environments.
- Controlling transistors and triacs.
- Switch-mode power supplies.
- PC/ Modem communication.
- Signal isolation.
- Power control.

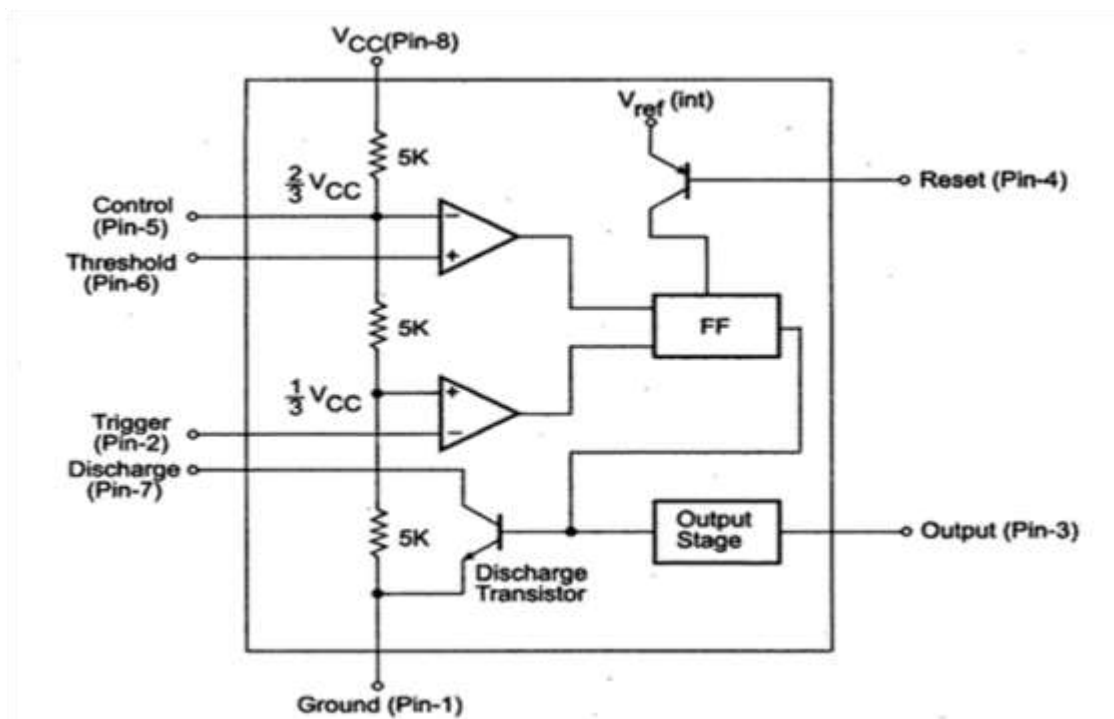
MULTIVIBRATORS USING IC-555

A multivibrator (like an oscillator) is a circuit with regenerative feedback, which produces a pulsed output. There are three basic types of multivibrator circuits:

- *Astable* – has no stable states, but switches continuously between two states. This action produces a train of square wave pulses at a fixed frequency.
- *Monostable* – one of the states is stable, but the other state is unstable (transient).
- *Bistable* – the circuit is stable in either state.

Timer IC-555:

Timer IC-555 is the one of the most commonly used general-purpose linear integrated circuits.

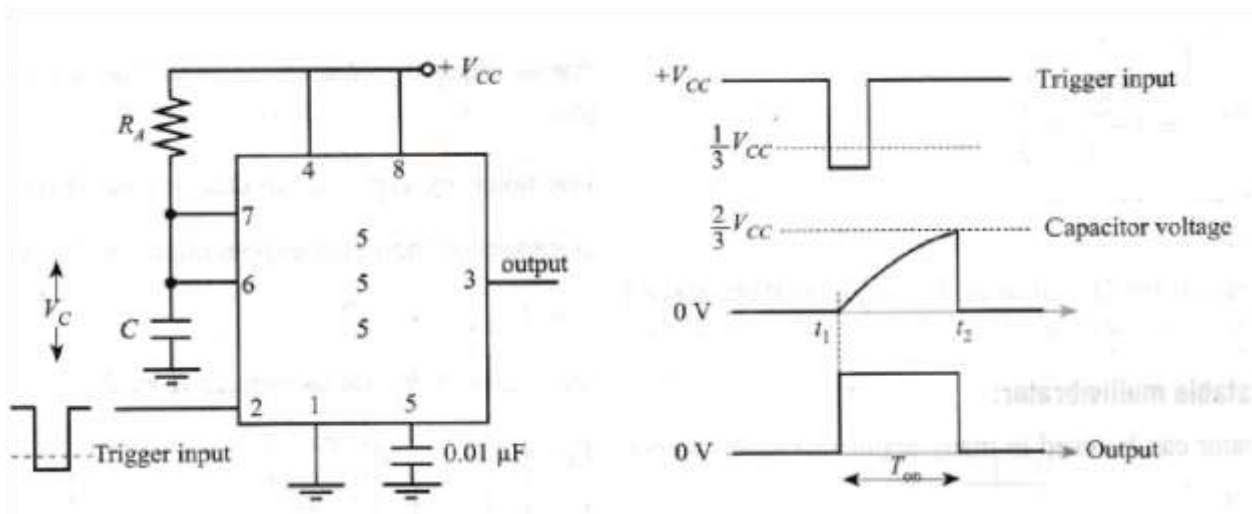
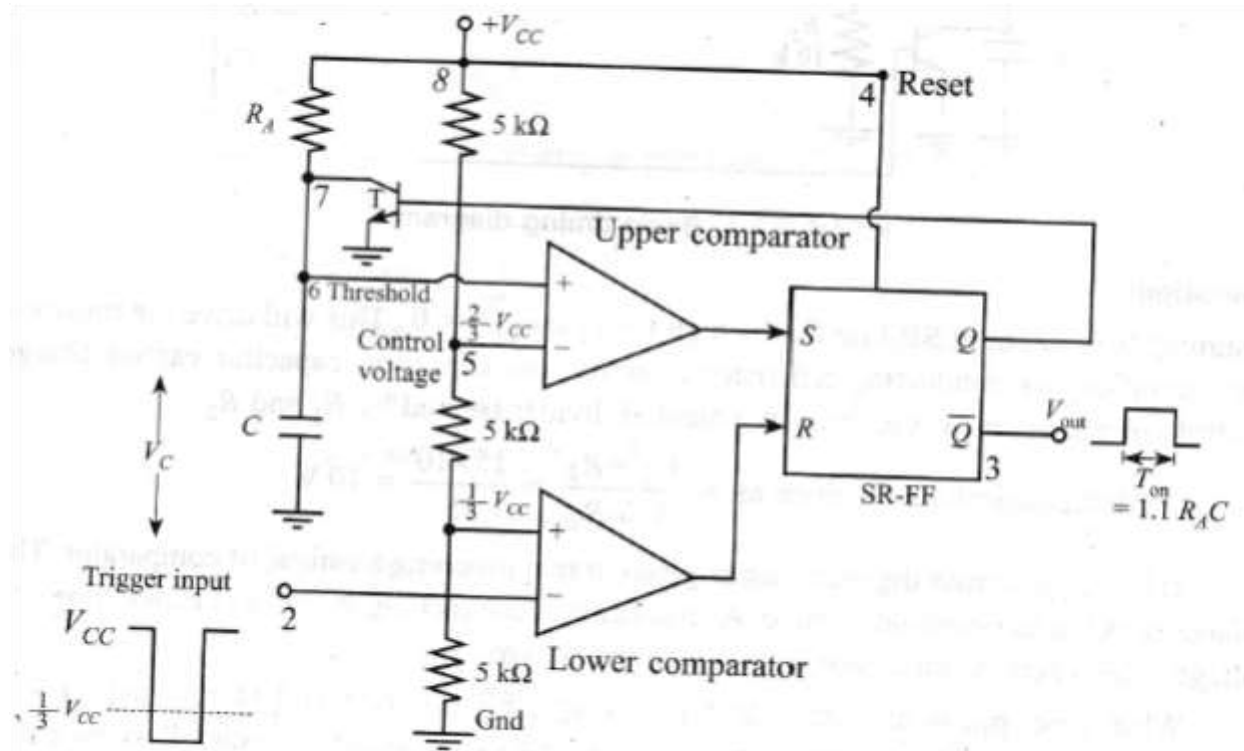


Internal Schematic of Timer IC-555

The Timer IC 555 comprises two Op-Amp comparators, a flip-flop, a discharge transistor, a reset transistor, three identical resistors and an output stage. The resistors set the reference voltage levels at the non-inverting input of the lower comparator and inverting input of the upper comparator at $+V_{CC}/3$ and $2V_{CC}/3$, respectively. The output of two comparator feed SET and RESET inputs of the Flip-Flop. This decided the logic state of its output and subsequently the final output. The Flip-Flops complementary outputs feed the output stage and the base of the discharge transistor. Hence, when the output is HIGH, the discharge transistor is OFF and when the output is LOW, the discharge transistor is ON.

MONOSTABLE MULTIVIBRATOR:

Monostable multivibrator using 555 timer IC is as shown in the following Figure. This 555 timer is called monostable multivibrator because it has only one stable state. Resistor R_A and capacitor C are components connected externally to the IC-555. Threshold voltage (6) and discharge (7) pins are connected to each other.



- Initially the SR-FF is set ($Q = 1$); transistor T is driven into saturation, and the capacitor is charged by the transistor. Therefore, the capacitor voltage $V_C = 0$, and also the output voltage $V_{out} = 0$ (as $Q = 1$).
- When the negative going trigger pulse (which should be more than $1/3 V_{CC}$) is applied at trigger input of lower comparator, the comparator output goes high; and SR-FF is reset ($Q = 0$), forcing Q to go high and transistor T turns off.

- As T is off, capacitor starts charging through R_A . Now, the output will remain high (from t_1 to t_2 , in waveform shown).
- At time t_2 ; the voltage across the capacitor V_C becomes more than $2/3 V_{CC}$ and upper comparator output goes high. This will set the SR-FF ($Q = 1$).
- Since, SR-FF output $Q = 1$; transistor T is ON, and hence, capacitor discharges, and also output goes low. The output remains low till the next trigger pulse is applied.

From the waveform of monostable multivibrator, it is clear that, the ON time T_{ON} of the output voltage is same as charging time of the capacitor.

Therefore, $T_{ON} \rightarrow$ is the time taken by capacitor to charge from 0 to $2/3 V_{CC}$.

The voltage across capacitor increases exponentially and is given by; $V_C = V_{CC} [1 - e^{-\frac{t}{RC}}]$

As capacitor charges through R_A ; let us replace R by R_A : Hence, $V_C = V_{CC} [1 - e^{-\frac{t}{R_A C}}]$

At $t = t_2$ (T_{ON}), the capacitor voltage (V_C) reaches $2/3 V_{CC}$:

$$V_C = \frac{2}{3} V_{CC} = V_{CC} [1 - e^{-\frac{T_{ON}}{R_A C}}] \quad e^{-\frac{T_{ON}}{R_A C}} = 1 - 2/3 = 1/3$$

Therefore, Or,

$$\text{Therefore, } T_{ON} = 1.1 R_A C$$

Applications:

A monostable multivibrator can be used in many applications, few important applications are

1. Frequency divider
2. Missing pulse detector
3. Pulse width modulator
4. Pulse position modulator etc.

Example 1:

For a monostable multivibrator time delay $T = 100$ ms and resistance $R = 105$ k Ω . Calculate the capacitor value.

Given: $T = 100$ ms, $R = 105$ k Ω

We know that the pulse width W is given by

$$W = 1.1RC = T_{on} \text{ (time delay)}$$

$$C = \frac{T_{on}}{1.1R} = \frac{100 \text{ ms}}{1.1 \times 105 \text{ k}}$$

$$\boxed{C = 0.865 \mu\text{F}}$$

Example 2:

Design a monostable multivibrator circuit using 555 timer to produce an output pulse of 20sec width.

The output pulse [T_{on}] is given as

$$T_{on} = 20 \text{ sec}$$

$$\text{WKT } T_{on} = 1.1RC \text{ Assuming } C = 150 \mu\text{F}$$

$$20 = 1.1 R \times 150 \mu\text{F}$$

$$R = \frac{20}{1.1 \times 150 \times 10^{-6}} = 121.21 \text{ k}\Omega$$

$$\boxed{R = 121.21 \text{ k}\Omega}$$

Example 3:

Find the resistive element value to generate $T = 10 \text{ ms}$ time delay, using 555 timer as a monostable multivibrator Assume $C = 0.47 \mu\text{F}$.

The on period of pulse is given by

$$T_{on} = 1.1RC$$

$$10 \times 10^{-3} = 1.1 \times R \times 0.47 \times 10^{-6}$$

$$R = \frac{10 \times 10^{-3}}{1.1 \times 0.47 \times 10^{-6}} = 19.34 \text{ k}\Omega$$

$$\boxed{R = 19.34 \text{ k}\Omega} \text{ We can choose standard value as } 18 \text{ k}\Omega.$$

Example 4:

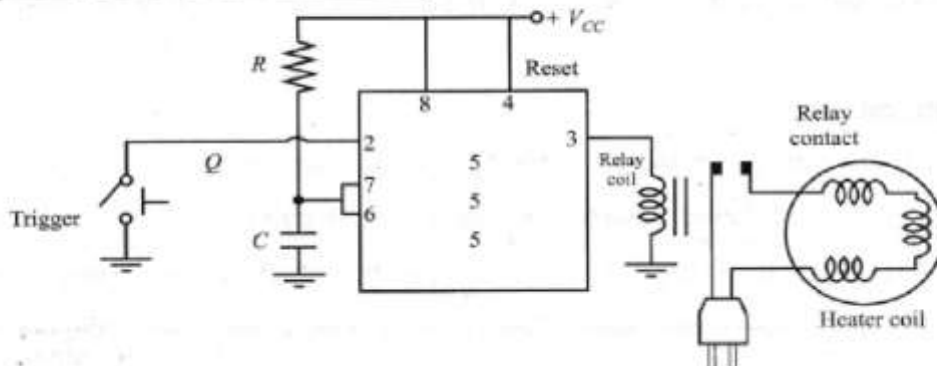
Design a timer that can turn on a heater immediately after pressing the button, and it should hold the heater in on-state for 10 seconds.

The relay coil should be energized for 10 seconds to hold heater on so T_{on} is 10 seconds and choosing $C = 47 \mu\text{F}$.

$$\text{WKT } T_{on} = 1.1RC$$

$$10 = 1.1RC$$

$$\boxed{R = \frac{10}{1.1 \times 47 \times 10^{-6}} = 193.42 \text{ k}\Omega}$$

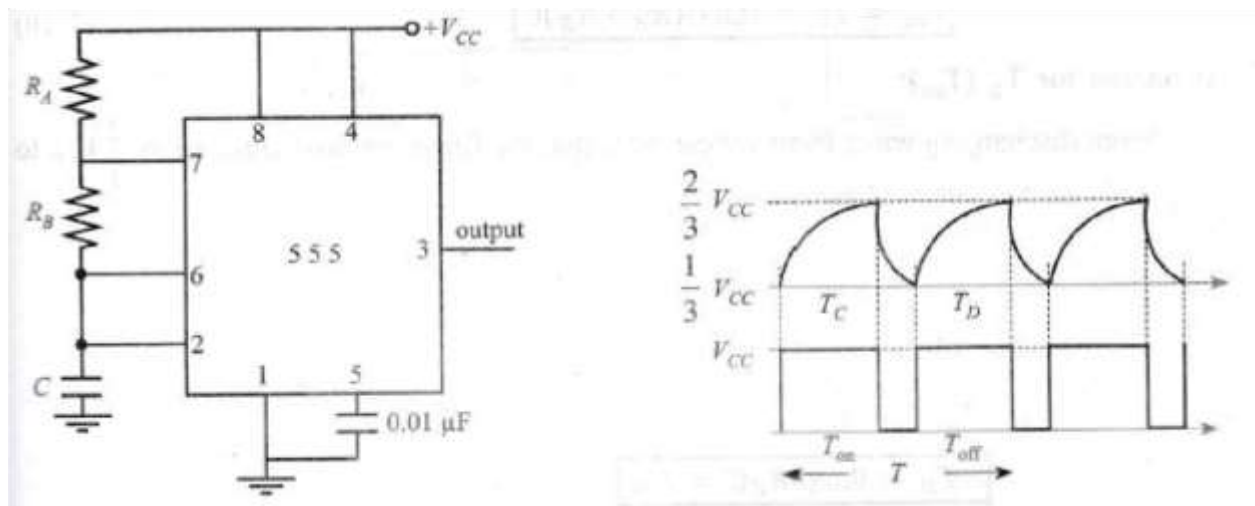
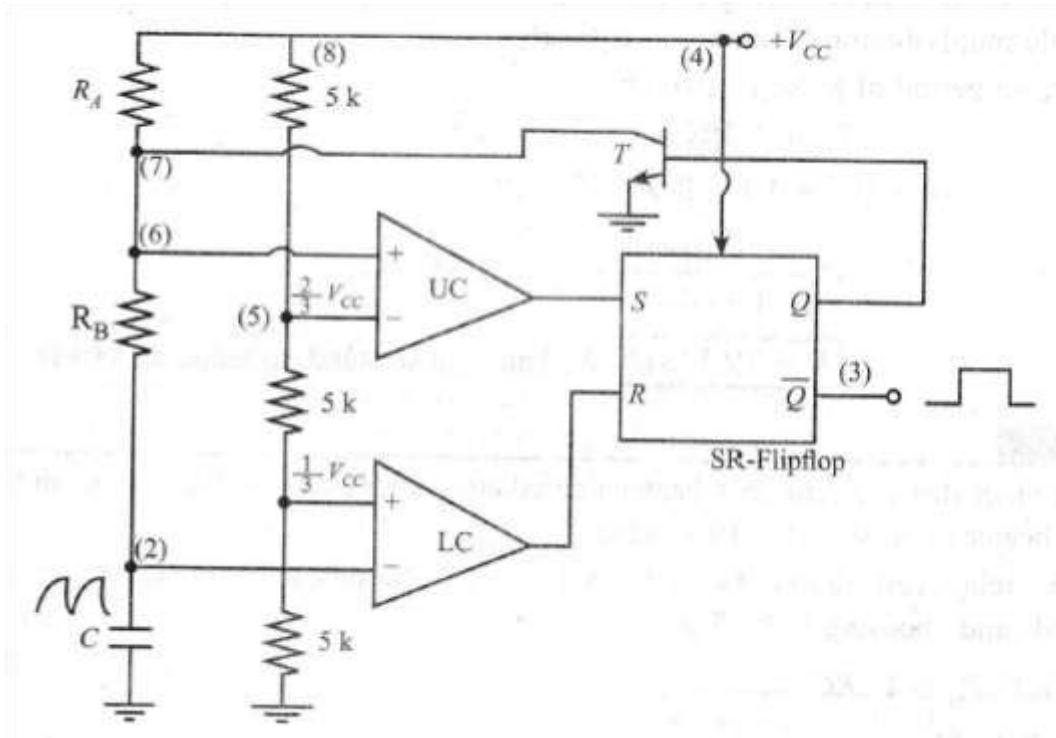


Monostable multivibrator used to turn-on relay

ASTABLE MULTIVIBRATOR:

An *astable multivibrator* does not have any stable state; it keeps changing its state from low to high and high to low. This multivibrator is also called *free running multivibrator* or *rectangular wave generator* circuit. *Astable multivibrator* does not require an external trigger pulse to change the state of the output.

The circuit configuration of an astable multivibrator is as shown in the following Figure.



To understand the operation, let us divide the circuit operation into two time interval T_{ON} and T_{OFF} .

ON time operation:

- At $t = 0$, the voltage on the capacitor $V_C = 0$, the same capacitor voltage is applied to both trigger point of lower comparator and threshold point of upper comparator. As capacitor voltage $V_C = 0$, which is less $1/3 V_{CC}$, the output of lower comparator goes high ($Q_1 = 1$) and $Q = 0$. This causes T to go off and capacitor starts charging through series resistors R_A and R_B . When capacitor voltages reaches $2/3 V_{CC}$, on time is terminated.

OFF time operation:

- As soon as V_C exceeds $2/3 V_{CC}$, the upper comparator output goes high and it will set the SR FF.

i.e., $S = 1$ and $R = 0$ and $Q = 1$ and $QQ = 0$. This will turn on transistor T , and output at pin (3) goes low.

- Now, the capacitor discharges through R_B , and through transistor T . The discharge time (also called off time (T_D); and it depends on the values of R_B and C . When capacitor voltage is $V_C = 1/3 V_{CC}$, lower comparator output goes high.
- This process of charging and discharging is continuous and hence circuit oscillates. The schematic diagram and waveforms are as shown in the Above Figure.

The output voltage waveform is the sum of charging and discharging periods (T_C , and T_D) of the capacitor.

∴ Period of one cycle $T = T_C + T_D$. Frequency can be written as $f = \frac{1}{T} = \frac{1}{T_C + T_D}$

Voltage across charging capacitor is given by (initial voltage on capacitor is zero);

$$V_C = V_{CC} [1 - e^{-\frac{t}{RC}}]$$

If there is some initial voltage present, the voltage expression for capacitor changes, and is given by;

$$V_C = V_F + (V_i - V_F)e^{-\frac{t}{RC}}$$

Where V_F - final voltage capacitor can reach, and V_i - is Initial voltage on capacitor.

During charging time T_C , the initial voltage on the capacitor is $V_i = 1/3 V_{CC}$, and the final voltage is $V_F = V_{CC}$. Also as charging takes place through both R_A and R_B , the above expression becomes;

$$\frac{2V_C}{3} = V_{CC} + (\frac{1}{3}V_{CC} - V_{CC})e^{-\frac{T_C}{(R_A + R_B)C}}$$

$$\frac{1}{3}V_{CC} = (\frac{2}{3}V_{CC})e^{-\frac{T_C}{(R_A + R_B)C}}$$

$$\text{Therefore, } T_{ON} = T_C = 0.693 (R_A + R_B) C$$

From discharging waveform; we can note that, the capacitor discharges from $2/3 V_{CC}$ to $1/3 V_{CC}$; and can be

expressed as: $\frac{1}{3}V_{CC} = 0 + (\frac{2}{3}V_{CC} - 0)e^{-\frac{T_D}{R_B C}}$

$$\text{Or, } \frac{1}{3}V_{CC} = (\frac{2}{3}V_{CC})e^{-\frac{T_D}{R_B C}}$$

$$\text{Therefore, } T_{OFF} = T_D = 0.693 R_B C$$

$$\text{Hence, Total period, } T = T_{ON} + T_{OFF} = 0.693 (R_A + 2R_B) C$$

$$\text{Frequency, } f = \frac{1}{T} = \frac{1.44}{(R_A + 2R_B)C}$$

$$\text{Duty Cycle, \%D} = [(\text{on time}) / (\text{total time})] * 100$$

$$\% D = \frac{(R_A + R_B)}{(R_A + 2R_B)} * 100$$

Applications:

A multivibrator can be used in many applications, few important applications are

1. Square-wave oscillator/ generator
2. Schmitt trigger using IC-555
3. Voltage controlled oscillator.

Example 1:

For an astable circuit $R_1 = 22 \text{ k}\Omega$, $R_2 = 30 \text{ k}\Omega$ and $C = 0.5 \text{ }\mu\text{F}$ Find on and off period of the output wave form shown in Figure 7-17.

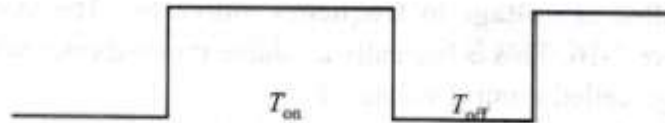


FIGURE 7-17

The on period corresponds to charging period of a capacitor and charging takes place through resistors R_1 and R_2 .

$$\begin{aligned}\therefore T_C = T_{on} &= 0.693(R_1 + R_2)C \\ &= 0.693(22 + 30) \times 10^3 \times 0.5 \times 10^{-6}\end{aligned}$$

$$\boxed{T_{on} = 18 \text{ msec}}$$

The off period corresponds to discharging time (T_D) of the capacitor and discharging taken place through R_2 only.

$$\begin{aligned}\therefore T_D = T_{off} &= 0.693R_2C \\ &= 0.693 \times 30 \times 10^3 \times 0.5 \times 10^{-6}\end{aligned}$$

$$\boxed{T_{off} = 10.395 \text{ msec}}$$

Example 2:

A 555 timer is configured to operate in astable mode with $R_A = 5 \text{ k}\Omega$, $R_B = 5 \text{ k}\Omega$ and $C = 0.01 \text{ }\mu\text{F}$. Determine the frequency of the output and duty cycle.

Given: $R_A = 5 \text{ k}\Omega$, $R_B = 5 \text{ k}\Omega$, $C = 0.01 \text{ }\mu\text{F}$

Frequency of oscillation for astable multivibrator is given by

$$f = \frac{1}{T} = \frac{1.45}{(R_A + 2R_B)C} = \frac{1.45}{(5 \text{ k} + 2(5 \text{ k}))0.01 \text{ }\mu\text{F}}$$

$$= \frac{1.45}{5 \times 10^3 + 2(5 \times 10^3) \times 0.01 \times 10^{-6}}$$

$$\boxed{f = 10.357 \text{ kHz}}$$

and Duty cycle $D = \frac{R_A + R_B}{R_A + 2R_B} = \frac{5 \times 10^3 + 5 \times 10^3}{5 \times 10^3 + 2(5 \times 10^3)}$

$$D = \frac{10 \times 10^3}{15 \times 10^3} = 66.66\%$$

$$\boxed{D = 66.66\%}$$

Example 3:

Design an astable multivibrator using 555 timer for a frequency of 2 kHz and a duty cycle of 75%. Assume $C_1 = 0.1 \text{ }\mu\text{F}$.

Given: $f = 2 \text{ kHz}$, duty cycle = 75%, $C = 0.1 \text{ }\mu\text{F}$

The on period $T_{\text{on}} = 0.693 (R_A + R_B) C$.

The off period $T_{\text{off}} = 0.693 R_B C$

Total period $T = T_{\text{on}} + T_{\text{off}} = 0.693(R_A + R_B) C$

\therefore Duty cycle D is given by

$$D = \frac{T_{\text{on}}}{T_{\text{on}} + T_{\text{off}}} = \frac{T_{\text{on}}}{T} = \frac{0.693 (R_A + R_B) \cancel{C}}{0.693 (R_A + 2R_B) \cancel{C}}$$

$$D = \frac{R_A + R_B}{R_A + 2R_B} = \frac{3}{4} = 0.75$$

$$\therefore 3(R_A + 2R_B) = 4(R_A + R_B)$$

$$3R_A + 6R_B = 4R_A + 4R_A + 4R_B$$

$$2R_B = R_A \quad \text{Or,} \quad R_B = \frac{1}{2} R_A$$

Substituting for R_B in expression for T

$$\therefore T = 0.693(R_A + R_B)C$$

$$T = 0.693 \left(R_A + \frac{1}{2} R_A \right) \times 0.1 \times 10^{-6}$$

$$T = 0.693 \left(\frac{3}{2} R_A \right) \times 0.1 \times 10^{-6}$$

$$\boxed{f = \frac{1}{T}}$$

$$1 \times 10^3 = f = \frac{1}{T} = \frac{1}{0.693(1.5R_A) \times 0.1 \times 10^{-6}}$$

$$R_A = \frac{1}{0.693(1.5)0.1 \times 10^{-6} \times 10^3}$$

$$\boxed{R_A = 9.6 \text{ k}\Omega}$$

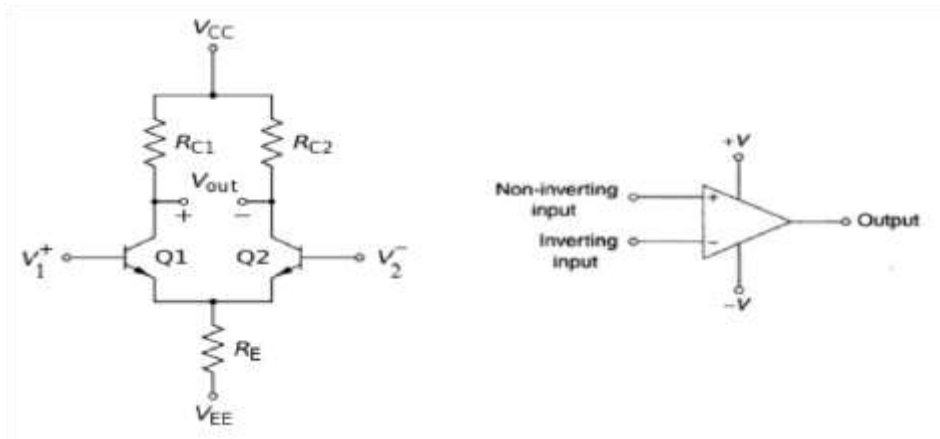
$$R_B = \frac{1}{2} R_A$$

$$= \frac{1}{2} \times 9.6 \times 10^3$$

$$\boxed{R_B = 4.8 \text{ k}\Omega}$$

OPERATIONAL AMPLIFIER (OP-AMP) APPLICATION CIRCUITS

An **Op-Amp** is a direct-coupled high gain, high bandwidth differential amplifier with very high value of input impedance and very low value of output impedance.



Basic Differential Amplifier & Circuit Representation of an OP-Amp

The ideal Op-Amp model was derived to simplify circuit calculations. The ideal Op-Amp model makes three assumptions:

1. Input resistance (impedance), $R_i = \infty$
 2. Output resistance (impedance), $R_o = 0$
 3. Open-loop (differential voltage) gain, $A_d = \infty$
- Based on these three assumptions,

other assumptions can be derived:

1. Since $R_i = \infty$, $I_i = I_{ni} = 0$
2. Since $R_o = 0$, $V_o = A_d * V_d$
3. Zero DC input and output offset voltages
4. Bandwidth and slew rate are also infinite, as no frequency dependencies are assumed.
5. Drift is also zero, as there is no changes in performance over time, temperature, power supply variations, and so on
6. Since output voltage depends only on differential input voltage, it rejects any voltage common to both inputs.
Hence, common mode gain = 0

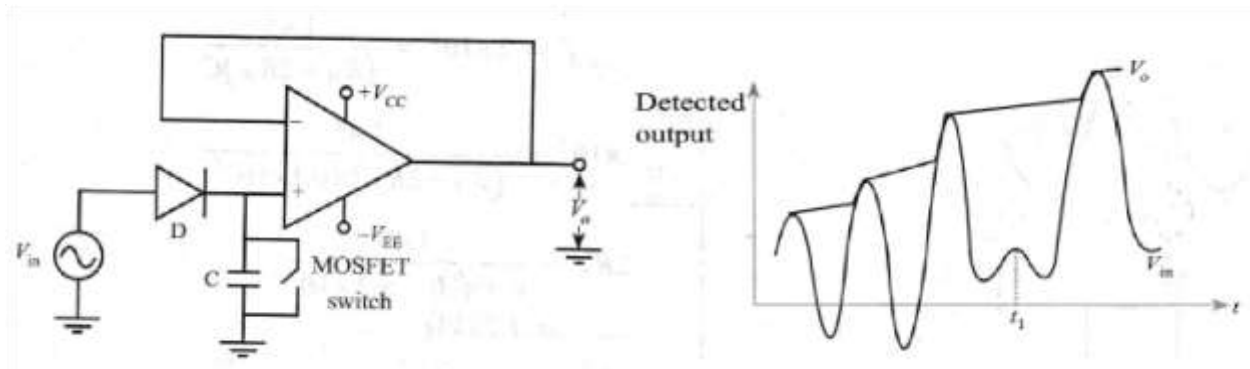
Open-loop gain is the differential voltage gain in the absence of any positive or negative feedback. Practical OpAmps have –

1. Input impedance can vary from hundred of kilo-ohms (for some low-grade Op-Amps) to tera-ohms (for high grade Op-Amps).
2. Output impedance may be in the range of 10 to 100 Ω
3. Open-loop gain in the range of 10,000 to 1,00,000
4. Bandwidth is limited and is specified by gain-bandwidth product
5. There may be some finite DC output (referred to as output offset voltage), even when both the inputs are grounded.

PEAK DETECTORS:

Ideal Op-Amp	Practical Op-Amp
Internal Impedance is infinite	Input Impedance range 100K Ω to 1000M Ω
Output Impedance is zero	Output impedance range from 10 Ω to 100 Ω
Open loop differential voltage gain is infinite	Open loop gain is in the range of 10,000 to 100,000
Bandwidth is infinite	Bandwidth is limited
DC input and output offset voltage is zero	Finite DC input and output offset voltage
Input differential voltage is zero	Finite differential voltage is finite

Peak detector detects holds the most positive value attained by the input signal. The following Figure shows peak detector circuit.

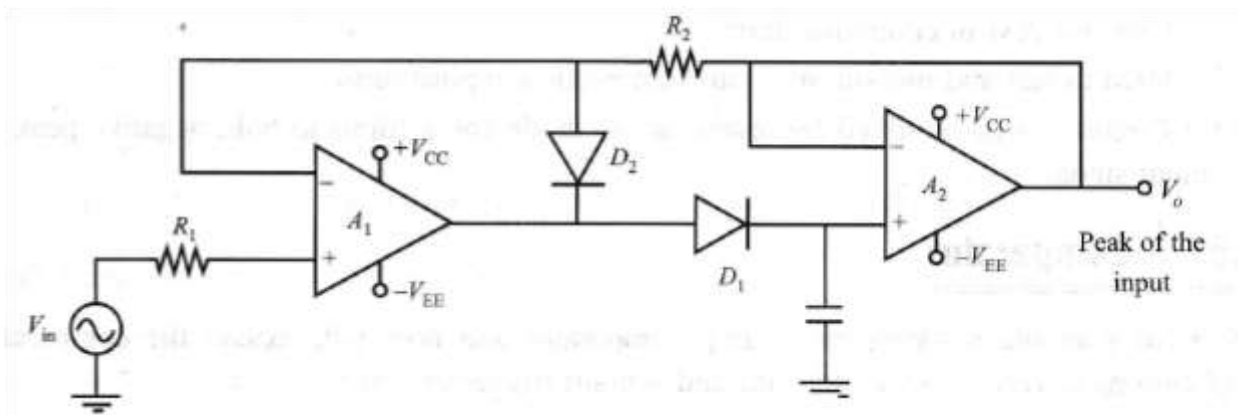


During positive half cycle of the input, D -conducts and capacitor charges to peak (highest) value of the input. Capacitor retains its charged value unless and until it discharges with a help of switch.

The op-amp is connected as a voltage follower and its output voltage will be equal the drop across capacitor which is positive peak value of the applied voltage and will remain that for long periods until next more higher peak occurs at the input. For negative cycle of input, the diode is reverse biased and capacitor retains its value.

Modified Peak Detector:

More sophisticated peak detector that buffers the signal source from the capacitor is shown in the following Figure.



As Op-Amp (A_1) is connected as voltage follower, the circuit presents very high impedance to the signal source. Op-Amp (A_2) acts as a buffer between the capacitor and the load. Output (V_o) at any given time is equal to the voltage on the capacitor which is nothing but, the peak value of the input occurred up to that time.

Whenever the input signal has higher peak than the present one, the capacitor charges up to new high input level. Whenever input level gets dropped, then capacitor retains the peak value of input, as diode D_1 gets reverse biased and diode D_2 prevents amplifier A_1 output from going into negative saturation.

To hold the negative peak of the input signal, reverse the diode connections in the above Figure.

Applications:

- Used for AM in communication
- Used in test and measurement instrumentation applications.

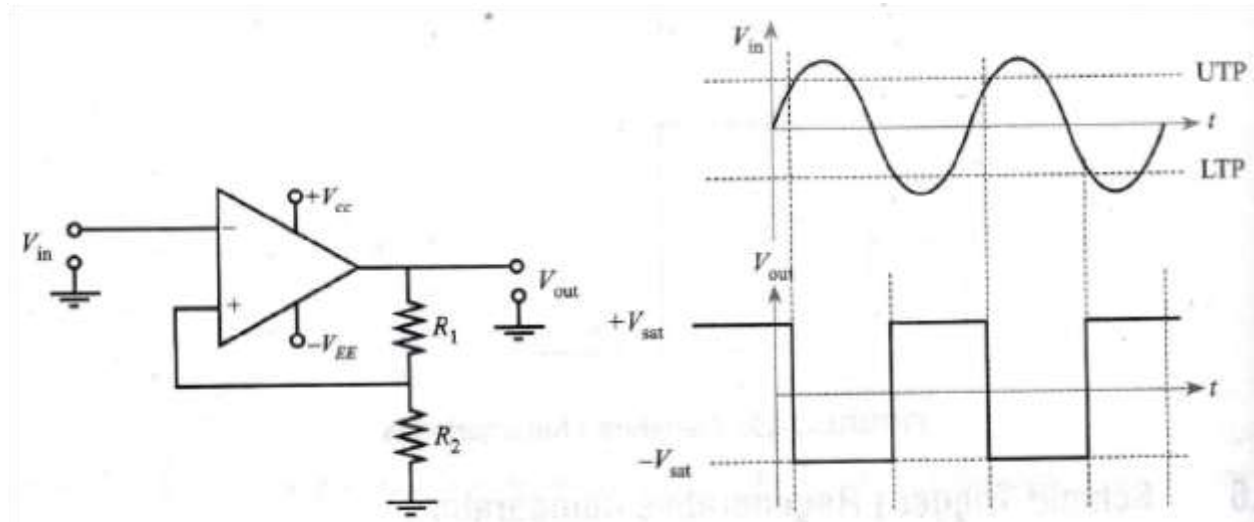
SCHMITT TRIGGER (REGENERATIVE) COMPARATOR:

A Schmitt trigger is a fast-operating voltage level detector.

Inverting Schmitt Trigger:

The input voltage V_{in} is applied to the inverting input terminal and the feedback voltage goes to the noninverting terminal. This means, the circuit uses positive voltage feedback (i.e., feedback voltage aids the input voltage).

If the input voltage at the inverting terminal is slightly positive than feedback voltage at the non-inverting terminal, the output voltage will be negative (negative saturation, $-V_{sat}$); and if the input voltage more negative than the reference feedback voltage, the output will be positive (positive saturation, $+V_{sat}$).



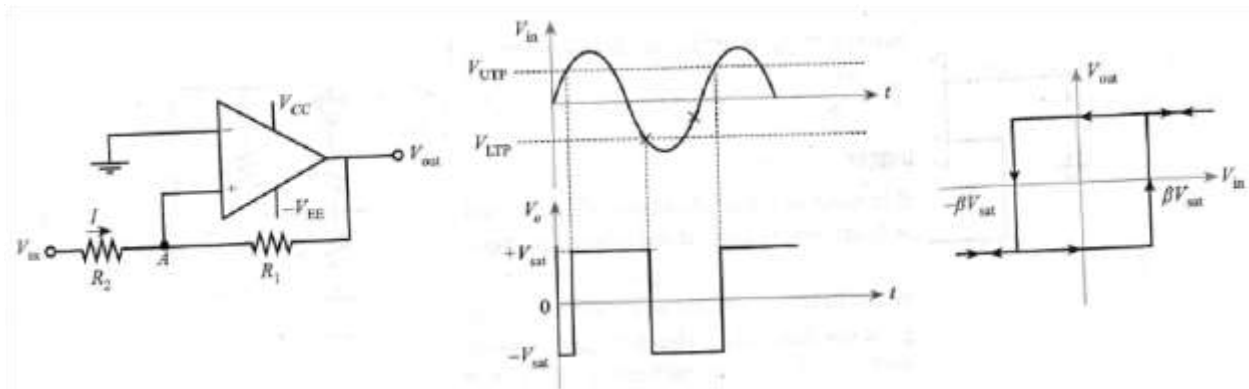
Hence, the voltage at the output switches from $+V_{sat}$ to $-V_{sat}$ or vice-versa; are called *Upper Trigger Point (UTP)* and *Lower Trigger Point (LTP)*. The difference between two trigger points is called *Hysteresis*. The upper and lower trigger points can be written as;

$$UTP = \frac{R_2}{(R_1 + R_2)} e V_{sat} \quad LTP = \frac{R_2}{(R_1 + R_2)} (-V_{sat})$$
$$V_{hys} = UTP - LTP = \frac{R_2}{(R_1 + R_2)} e V_{sat} - \frac{R_2}{(R_1 + R_2)} e (-V_{sat}) = 2 \left(\frac{R_2}{R_1 + R_2} \right) V_{sat} = 2\beta V_{sat}$$
$$\beta = \frac{R_2}{R_1 + R_2}$$

Non-Inverting Schmitt Trigger:

The input voltage V_{in} is applied to the non-inverting input terminal and the feedback voltage also goes to the non-inverting terminal. The inverting terminal is grounded..

Initially, assume that the output is in the negative saturation ($-V_{sat}$). Then the feedback voltage is also negative. This feedback voltage will hold the output in negative saturation, until the input voltage becomes positive enough to make voltage positive.



Let V_A is the voltage at point A. Hence, $V_A = IR_2$.

Since no current passes through the Op-Amp, entire current flows through R_2 .

$$\text{Therefore, } I = \frac{V_0}{R_1} = \frac{+V_{sat}}{R_1}$$

When V_{in} becomes positive and its magnitude becomes greater than $(R_2/R_1)V_{sat}$, then the output switches to $+V_{sat}$. Therefore, the UTP at which the output switches to $+V_{sat}$ is given by;

$$UTP = \frac{R_2 V_{sat}}{R_1}$$

Similarly, when V_{in} becomes negative and its magnitude becomes greater than $(R_2/R_1)V_{sat}$, then the output switches to $-V_{sat}$. Therefore, the LTP at which the output switches to $-V_{sat}$ is given by;

$$LTP = -\frac{R_2 V_{sat}}{R_1}$$

$$V_{hys} = UTP - LTP = 2 \left(\frac{R_2}{R_1} \right) V_{sat} = 2\beta V_{sat}$$

$$\beta = \frac{R_2}{R_1}$$

Applications of Schmitt Trigger:

Schmitt trigger is used in many applications, where level needs to be sensed. Hysteresis is used to reduce the multiple transitions that can occur around.

- Digital to analog conversion ○
- Level detection ○ Line reception.

Example 2:

Design a Schmitt trigger whose threshold voltages are ± 5 V. Draw its wave forms.

Choosing op-amp with $V_{sat} = \pm 13.5$ V with supply ± 15 V.

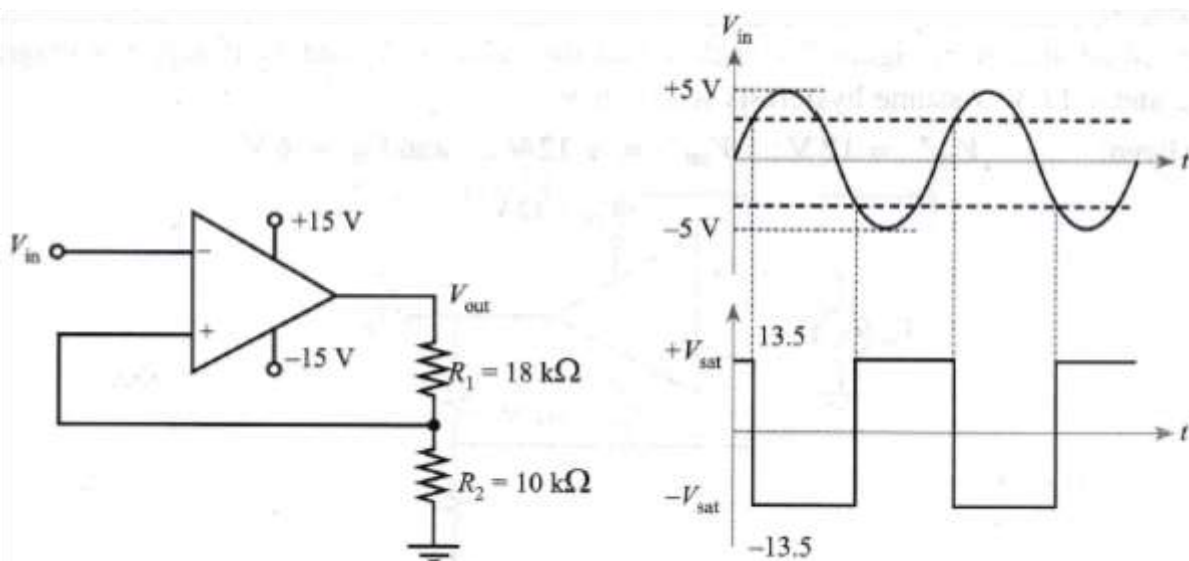
$$V_{UTP} = +5 \text{ V}$$

$$\text{Now } V_{UTP} = \frac{R_2}{R_1 + R_2} \cdot V_{sat} \quad \text{i.e., } 5 = \frac{R_2}{R_1 + R_2} \times 13.5$$

$$\therefore R_1 + R_2 = 2.7 R_2 \quad \text{i.e. } R_1 = 1.7 R_2$$

$$\text{Choose } R_2 = 10 \text{ k}\Omega \quad \therefore R_1 = 17 \text{ k}\Omega \text{ (Use } 18 \text{ k}\Omega)$$

The designed circuit with waveform are shown below.

**Example 3:**

For the circuit shown, $R_2 = 120 \Omega$ and $R_1 = 51 \text{ k}\Omega$. Determine the threshold voltages, if power supply applied to the op-amps are $+15$ V and -15 V.

Given

$$V_{sat}^+ = +V_{CC} = +15 \text{ V.}$$

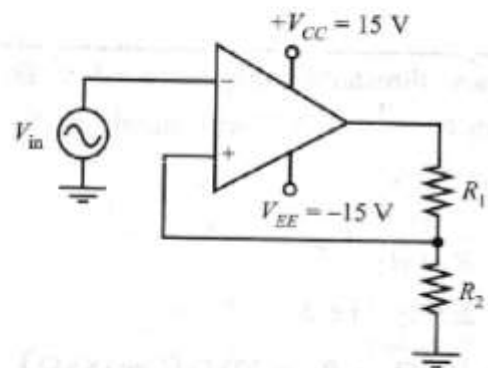
$$V_{sat}^- = -V_{EE} = -15 \text{ V.}$$

$$R_1 = 51 \text{ k}\Omega, R_2 = 120 \Omega$$

WKT

$$V_{UTP} = \frac{V_{sat}^+ R_2}{R_1 + R_2} = \frac{15 \times 120}{51 \times 10^3 + 120} = 35.2 \text{ mV.}$$

$$V_{LTP} = \frac{V_{sat}^- R_2}{R_1 + R_2} = \frac{-15 \times 120}{51 \times 10^3 + 120} = -35.2 \text{ mV}$$



$$2 = 28 \frac{R_2}{R_1 + R_2} \quad 28R_2 = 2R_1 + 2R_2 \quad 26R_2 = 2R_1 \quad \text{i.e., } R_1 = 13R_2$$

$$\text{Let } R_2 = 10 \text{ k}\Omega \quad R_1 = 13 \times 10 \text{ k} = 130 \text{ k}\Omega$$

Example 6:

Design schmitt trigger circuit with $UTP = -2$ and $LTP = -4$ V. $V_{sat} = 14$ V.

$$\text{We have, } UTP - LTP = \frac{2R_2}{R_1 + R_2} V_{sat}$$

$$-2 + 4 = 2 = \frac{2R_2 V_{sat}}{R_1 + R_2} = \frac{2R_2}{R_1 + R_2} \times 14$$

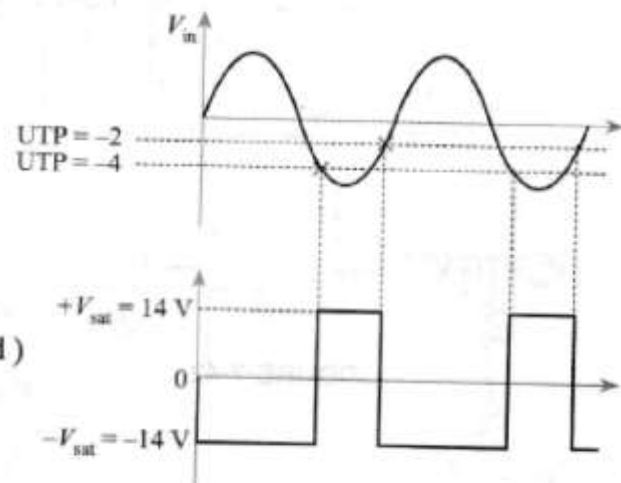
$$2 = \frac{28R_2}{R_1 + R_2} \Rightarrow R_1 = 13R_2$$

$$\text{Let } R_2 = 10 \text{ k} \quad \therefore R_1 = 130 \text{ k}\Omega \text{ (120 k std)}$$

Also WKT

$$UTP + LTP = \frac{2R_1 V_{Ref}}{R_1 + R_2}$$

$$V_{Ref} = \frac{(R_1 + R_2)(UTP + LTP)}{2R_1} \quad V_{Ref} = 3.27 \text{ V.}$$



Example 7:

For the schmitt trigger $R_1 = 3 \text{ k}\Omega$ and $R_2 = 1 \text{ k}\Omega$ calculate the V_{UTP} , V_{LTP} and V_H . Assume saturation voltages as ± 12 V.

$$\text{Given: } V_{sat}^+ = 12 \text{ V; } V_{sat}^- = -12 \text{ V; } R_1 = 2 \text{ k}\Omega; \quad R_2 = 3 \text{ k}\Omega$$

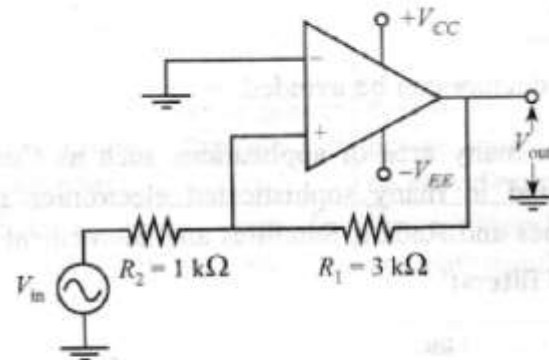
WKT

$$V_{UTP} = \frac{(V_{sat}^+) R_1}{R_2} = \frac{(12) \times 1 \text{ k}}{3 \text{ k}} = 4 \text{ V}$$

$$V_{LTP} = \frac{-(V_{sat}^-) R_1}{R_2} = \frac{-(12) \times 1 \text{ k}}{3 \text{ k}} = -4 \text{ V}$$

The hysteresis width is given by

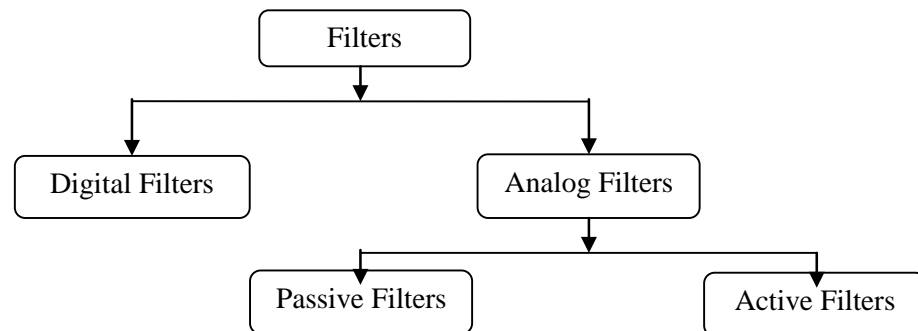
$$V_H = \frac{R_1}{R_2} [V_{sat}^+ - V_{sat}^-] = \frac{1 \text{ k}}{3 \text{ k}} [12 - (-12)] = \frac{24}{3} = 8 \text{ V.}$$



Filter is a frequency selective circuit commonly used in signal processing that passes signal of specified range of frequencies and blocks the signals of frequencies outside the band. Active filters are attractive due to their –

- Flexibility in gain control
- Small component size
- No loading Problem
- Pass band gain
- Use of the inductors can be avoided

Filters are useful in many areas of applications, such as Communication and Signal Processing. They are found in electronic systems like Radio, Television, Telephones, Radars, satellites, and Biomedical instruments.



Broader Classification of Filters

Passive filters work for high frequencies; but at audio frequencies, the inductors become problematic, as they are large, heavy, high power dissipation, and expensive.

Active filters use Op-Amp as the active element, resistors and capacitors as passive elements.

SNo.	Passive Filters	Active Filters
1	Filters with only components like resistors, capacitors, and inductors are known as passive filters.	Filters with components such as Op-Amps, transistors, and other active elements are known as active filters.
2	Passive filters do not require an external power source for operation; incapable of providing power gain.	Active filters require an external power supply for operation; capable of providing power gain.
3	Better stability and can withstand large currents.	Oscillations and noise will be generated due to feedback loops.
4	A passive filter has no frequency	Due to active elements, active filters have
	limitations.	frequency limitations.

5	Passive filters circuits are bulky/ heavy due to the presence of inductors; they consume more power and operate with limited speed.	Active filters circuits are more compact, less heavy; and operate with high speed.
6	Difficult to fabricate in IC form and usually designed using discrete components.	Can be fabricated in IC form and usually designed using discrete components.

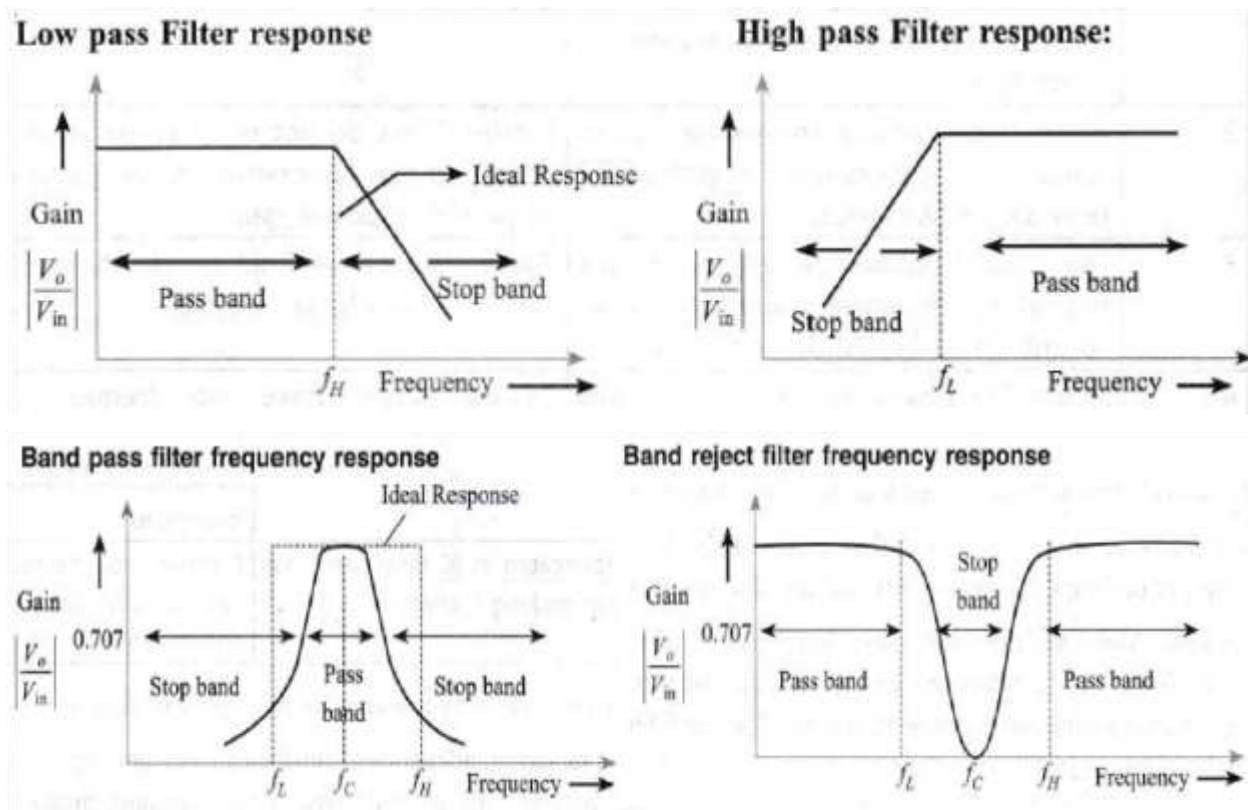
Active filters offer the following advantages over Passive filters:

- Gain and frequency adjustment flexibility
- No loading problem & No insertion loss
- Size and weight
- Cost.

Most commonly used active filters are –

- Low-pass filter, High-pass filter, Band-pass filter, Band-stop filter (Band-reject filter), and Allpass filter.

Frequency Response:

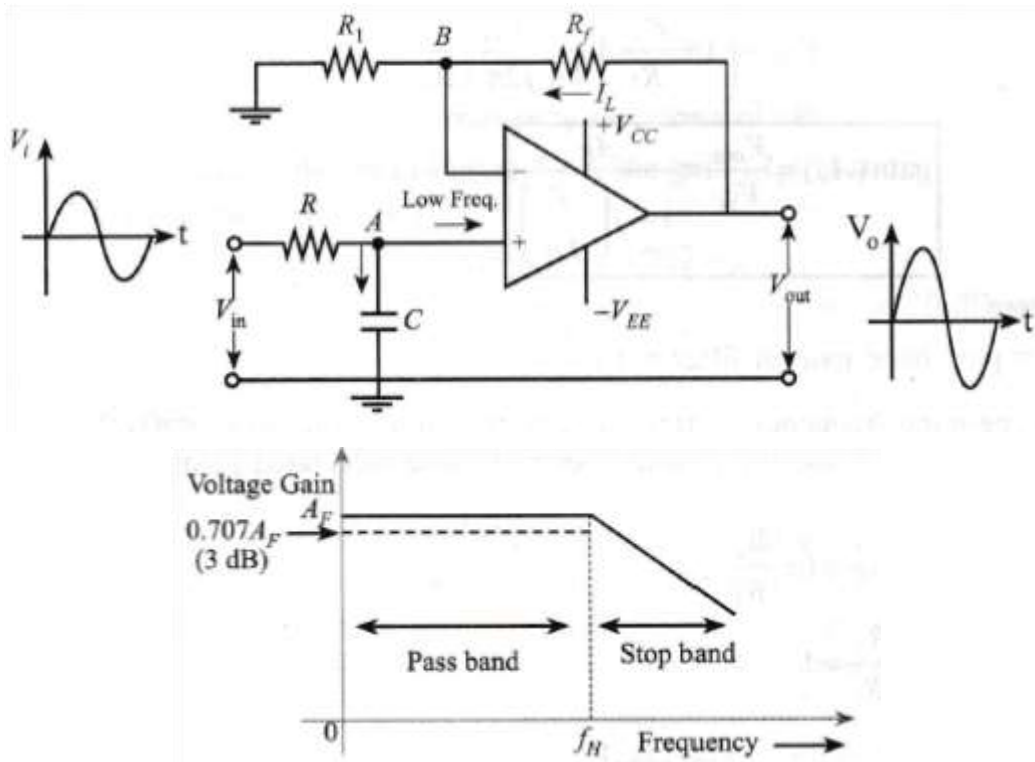


Design:

An active filter generally uses Op-Amp. Op-Amp has very high input impedance and low output impedance. The gain is determined by the resistive network in the feedback loop.

First Order Active Low-Pass Filter (LPF):

The first-order low-pass butter worth filter consists of a single RC filter stage, providing a low frequency path to the non-inverting input of an Op-Amp. The circuit diagram and the frequency response of the circuit is given below:



- From the graph; the gain (A_F) is almost constant for the frequency range: $0 < f < f_H$.
- At cut-off frequency, $f = f_H$, the gain is $0.707A_F$.
- After cut-off frequency f_H , the gain decreases at the rate of 20 dB/decade.
- The cut-off frequency is given by: $f_H = \frac{1}{2\pi RC}$
- Pass band gain is given by: $A_F = 1 + \frac{R_f}{R_1}$

Some applications of low-pass filters are –

- Low-pass filters are used in Audio amplifiers
- LPFs are used in equalizers or speakers to reduce the high frequency noise.

Example 1:

Design a first order low pass filter with cut-off frequency of 2.2 kHz and with pass band gain of 2.

Given: cut-off frequency $f_H = 2.2 \text{ KHz}$.

Let us choose $C = 0.01 \mu\text{F}$.

WKT
$$f_H = \frac{1}{2\pi RC}$$

$$\Rightarrow R = \frac{1}{2\pi f_H C} = \frac{1}{2\pi \times 2.2 \times 10^3 \times 0.01 \times 10^{-6}}$$

$$R = 7.233 \text{ k}\Omega$$

For frequency scaling, use the potentiometer of value calculated below.

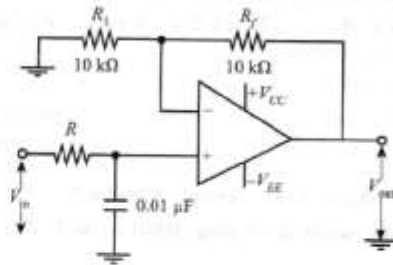
W.K.T

$$A_F = 1 + \frac{R_f}{R_1} = 2$$

$$\frac{R_f}{R_1} = 1$$

Therefore $R_f = R_1 = 10 \text{ k}\Omega$ (Take).

The low pass filter circuit with designed values is as follows:



Example 2:

Determine the value of the resistance required for the low pass filter with cut-off frequency 30 k rad/sec and capacitor value $= 0.001 \mu\text{F}$.

Given $\omega_H = 30 \text{ k rad/sec}$ and $C = 0.001 \mu\text{F}$.

WKT $\omega_H = 2\pi f_H$

and

$$f_H = \frac{1}{2\pi RC}$$

$$\therefore \omega_H = 2\pi \times \frac{1}{2\pi R \times 0.001 \times 10^{-6}} = 30 \times 10^3$$

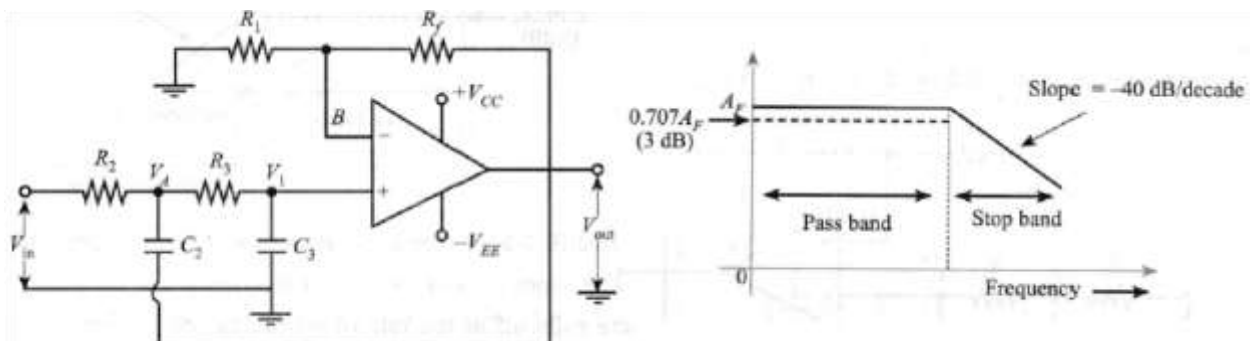
$$R = \frac{1}{30 \times 10^3 \times 0.001 \times 10^{-6}}$$

$$= \frac{1}{30 \times 10^3 \times 10^{-3} \times 10^{-6}} = 0.33 \times 10^5$$

$$\boxed{R = 33 \text{ k}\Omega}$$

Second Order Low-Pass Filter:

First order filter can be converted to second order filter by adding an extra RC-network, as shown in the following Figure. The frequency response of second order low-pass filter is same as the first order lowpass filter except that the gain at the stop band rolls off at the rate of 40 dB/decade .



- After cut-off frequency f_H , the gain decreases at the rate of 40 dB/decade .

- The cut-off frequency is given by: $f_H = \frac{1}{2\pi\sqrt{R_2R_3C_2C_3}}$. ○ If $R_2 = R_3 = R$ & $C_2 = C_3 = C$; then

$$f_H = \frac{1}{2\pi RC}$$

$$A_F = 1 + \frac{R_f}{R_i}$$

- Pass band gain is given by:

Example 1:

Determine the values of Resistance required for second order low pass Butter worth filter having cut-off frequency as 15 k rad/sec with capacitor value as 0.01 μ F.

Given: $\omega_H = 15$ k rad/sec, $C = 0.01$ F.

From equation (7.38)

$$f_H = \frac{1}{2\pi\sqrt{R_2R_3C_2C_3}}$$

But $R_2 = R_3 = R$ and $C_2 = C_3 = C$ and C as 0.01 μ F

$$f_H = \frac{1}{2\pi\sqrt{(R)^2 \times (0.01 \times 10^{-6})^2}} = \frac{1}{2\pi \times R \times 0.01 \times 10^{-6}} \quad (*)$$

$$\text{and } \omega_H = 2\pi f_H = 15 \text{ k rad/sec} = 2\pi \times f_H = 15 \times 10^3 \quad f_H = \frac{15 \times 10^3}{2\pi} = 2.3870 \text{ kHz}$$

on substituting f_H in equation(*), it becomes

$$2.3870 \times 10^3 = \frac{1}{2\pi \times R \times 0.01 \times 10^{-6}} \quad R = 6.67 \text{ k}\Omega \quad R_2 = R_3 = 6.67 \text{ k}\Omega$$

Example 2:

Design a second order low pass filter with cut-off frequency of 10 kHz and unity gain at low frequency. Also calculate the voltage transfer function magnitude at 15 kHz for the filter.

$$\text{Given: Cut-off frequency } f_H = \frac{1}{2\pi RC}$$

$$\therefore RC = \frac{1}{2\pi f_H} = \frac{1}{2\pi \times 10 \times 10^3} = 15.92 \times 10^{-6}$$

$$\text{Let } R = 100 \text{ k}\Omega, \text{ then } C = \frac{15.92 \times 10^{-6}}{100 \times 10^3} = 0.159 \text{ pF}$$

Therefore $C_2 = 0.159$ pF and $C_3 = 0.159$ pF

The voltage transfer function is given by

$$\frac{V_o(s)}{V_{in}(s)} = \frac{A_F}{\sqrt{1 + \left(\frac{f}{f_H}\right)^4}} = \frac{1}{\sqrt{1 + \left(\frac{15 \times 10^3}{10 \times 10^3}\right)^4}} = 0.406$$

Example 3:

Design a second order low pass butter worth filter for the cut-off frequency $f_H = 200$ Hz and draw the circuit diagram.

Given : $f_H = 200$ Hz.

Choose $R_2 = R_3 = R$ and $C_2 = C_3 = C = 0.1 \mu\text{F}$

Now, let us find value of R

$$f_H = \frac{1}{2\pi RC}$$

$$200 = \frac{1}{2\pi \times R \times 0.1 \times 10^{-6}}$$

$$\boxed{R = 7.96 \text{ k}\Omega.}$$

For second order filter

$$R_f = 0.586R,$$

i.e $A_F = 1 + \frac{R_f}{R_1} = 1.586.$

choosing $R_1 = 10 \text{ k}\Omega$

$$1 + \frac{R_f}{R_1} = 1.586$$

$$1 + \frac{R_f}{10 \text{ k}\Omega} = 1.586$$

$$\frac{R_f}{10 \text{ k}\Omega} = 0.586$$

$$R_f = 5.86 \text{ k}\Omega$$

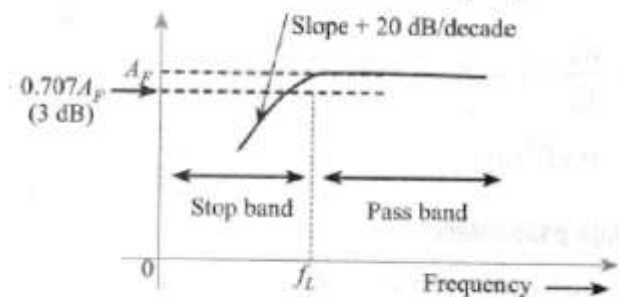
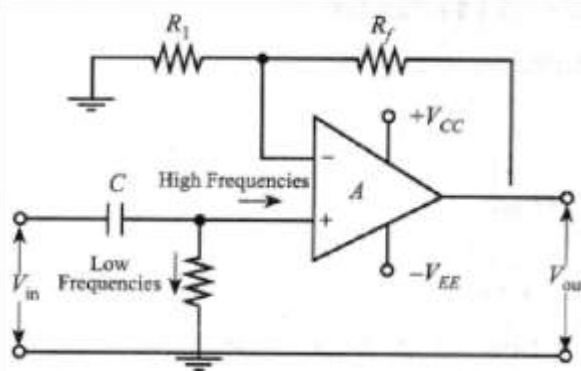
For adjustment use $10 \text{ k}\Omega$ pot.

High-Pass Butter Worth Filter:

High-pass filters passes higher frequency signals, attenuating all signals below cut-off frequency, f_L .

First Order High-Pass Butter Worth Filter:

The filter circuit consists of a passive filter followed by a non-inverting amplifier.



- At low frequency: $f < f_L$, $\frac{V_0}{V_{in}} < A_F$. A_F increases at the rate of 20 dB/decade till $f = f_L$.
- At cut-off frequency, $f = f_L$, the gain is $0.707A_F$.
- At very high frequency $f > f_L$, $\frac{V_0}{V_{in}} = A_F$ is constant.
- The cut-off frequency is given by: $f_L = \frac{1}{2\pi RC}$
- Pass band gain is given by: $A_F = 1 + \frac{R_f}{R_1}$

Example 1:

Design a first order high pass filter with a cut-off frequency of 10 kHz with pass band gain of 2.

Given: $f_L = 10 \text{ kHz}$ (given)

Choose $C < 1 \text{ } \mu\text{F}$.

Let $C = 0.01 \text{ } \mu\text{F}$

Calculate $R = \frac{1}{2\pi f_c}$

$$R = \frac{1}{2\pi \times 10 \times 10^3 \times 0.01 \times 10^{-6}}$$

$$\boxed{R = 1.5913 \text{ k}\Omega}$$

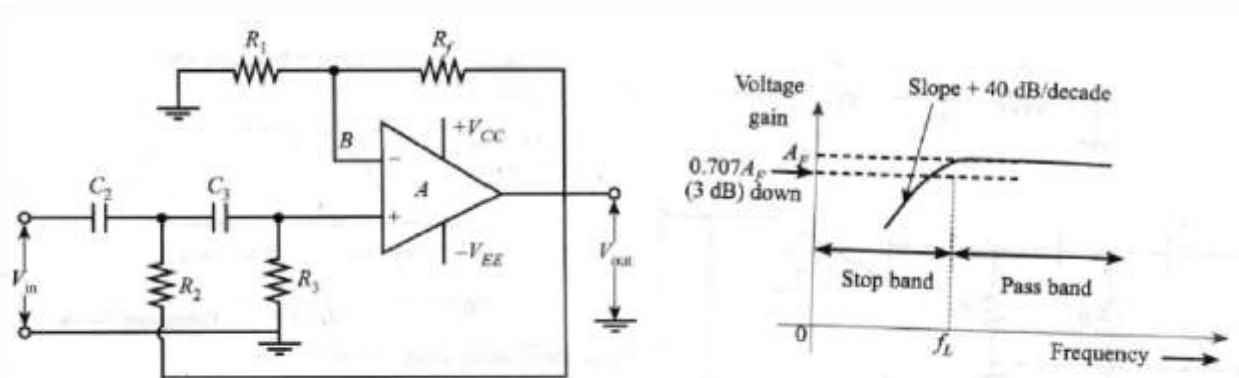
Step 4: Gain $A_F = 2 = 1 + \frac{R_f}{R_1}$

$$\therefore \frac{R_f}{R_1} = 1 \Rightarrow R_f = R_1$$

Choosing $R_1 = 10 \text{ k}\Omega$ (say).

Second Order High-Pass Filter:

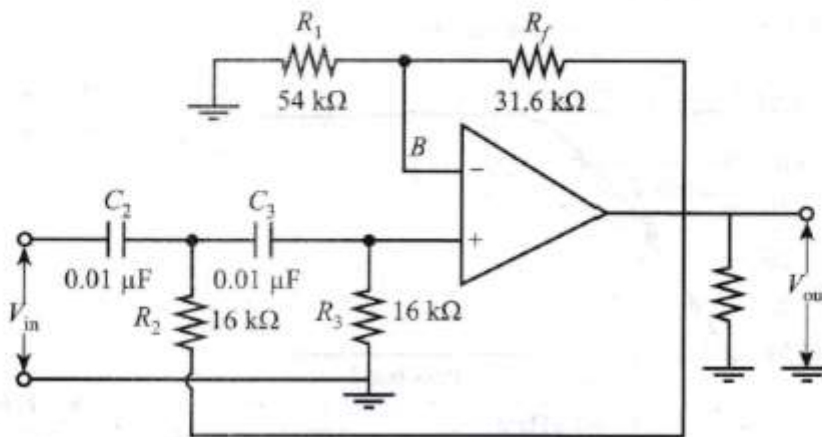
A first order high-pass filter can be converted into a second order high-pass filter by using an extra RC network in the input side. The frequency response of second order high-pass filter is same as the first order high-pass filter except that the gain at the stop band rolls off at the rate of 40 dB/decade.



- The cut-off frequency is given by: $f_L = \frac{1}{2\pi\sqrt{R_2 R_3 C_2 C_3}}$
 - If $R_2 = R_3 = R$ & $C_2 = C_3 = C$; then $f_L = \frac{1}{2\pi RC}$
- Pass band gain is given by: $A_F = 1 + \frac{R_f}{R_1}$

Example 1:

For the circuit shown in Figure find (a) lower cut-off frequency (b) pass band gain



Given: $R_2 = R_3 = 16 \text{ k}\Omega$ and $C_2 = C_3 = 0.01 \text{ }\mu\text{F}$

(a) The lower cut-off frequency for second order high pass filter is given by

$$f_L = \frac{1}{2\pi\sqrt{R_2 R_3 C_2 C_3}} = \frac{1}{2\pi\sqrt{(16 \times 10^3)^2 (0.01 \times 10^{-6})^2}} = 1 \text{ kHz}$$

$$\boxed{f_L = 1 \text{ kHz}}$$

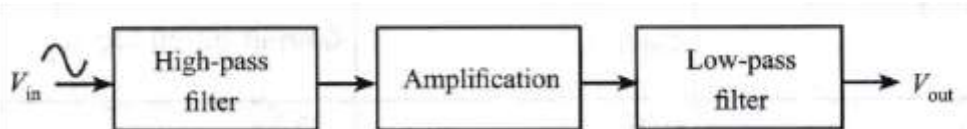
b) The pass band gain is

$$A_F = 1 + \frac{R_f}{R_1} = 1 + \frac{31.6 \times 10^3}{54 \times 10^3} = 1.586$$

$$\boxed{A_F = 1.586}$$

Active Band-Pass Filter:

Active band-pass filters provide an effective means of making a filter to pass only a given band of frequencies. An active band-pass filter can be constructed by cascading a single low-pass filter with a single high-pass filter, as shown below:



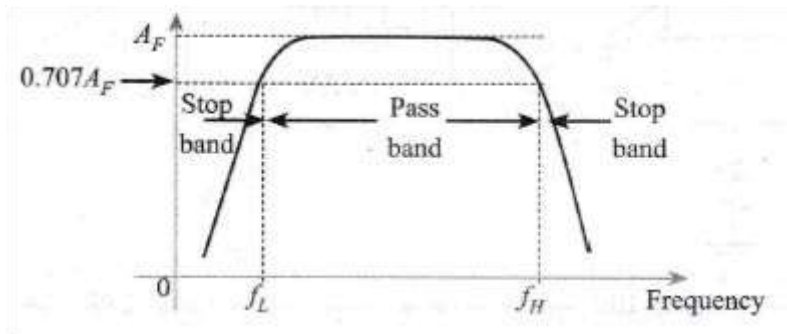
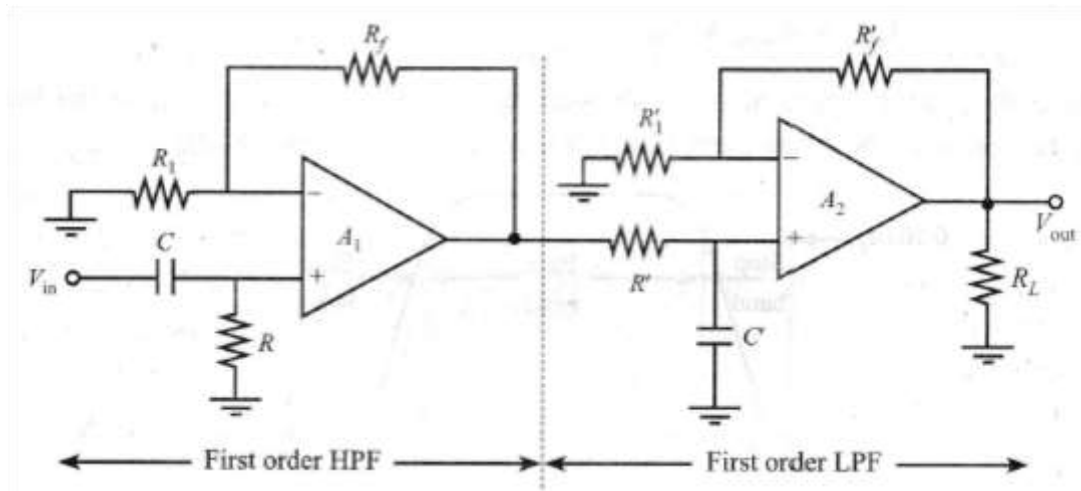
The cut-off frequency of the low-pass filter is higher than the cut-off frequency of the high-pass filter; and the difference between these frequencies at the -3dB point will give the 'bandwidth' of the band-pass filter.

A band-pass filter can be characterized by Quality factor (Q). The relation between Q , 3dB bandwidth, and the centre frequency, f_c , is given by; $Q = \frac{f_c}{BW} = \frac{f_c}{f_H - f_L}$

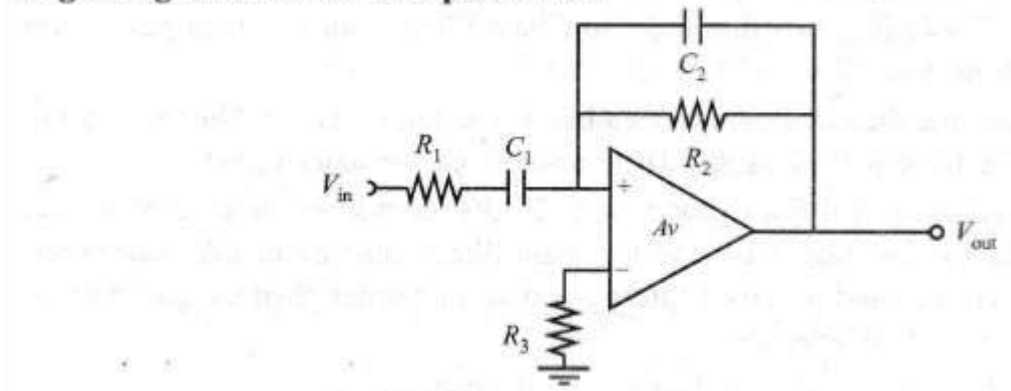
Wide Band-Pass Filter:

A low Q -filter will have a wide-pass-band; i.e., with $Q < 10$. It has wide flat response over the range of

frequencies and bandwidth is large.



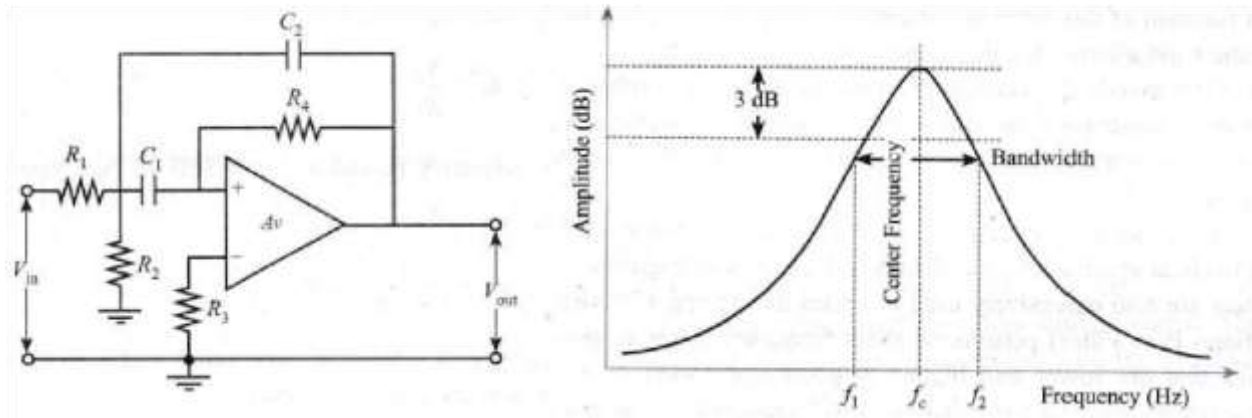
Single stage first order Bandpass filter:



Narrow Band-Pass Filter:

A high Q -filter will have a narrow-pass-band; i.e., with $Q > 10$. It has a sharp bell type response, with high gain and high selectivity.

- $f_c = \frac{1}{2\pi\sqrt{R_1 R_2 C_1 C_2}}$
- $Q_{BP} = \frac{f_c}{BW_{3dB}} = \frac{1}{2}\sqrt{\frac{R_1}{R_2}}$
- Maximum Gain, $A_v = -\frac{R_2}{2R_1} = -2Q^2$



Applications of Band-Pass Filters (BPF):

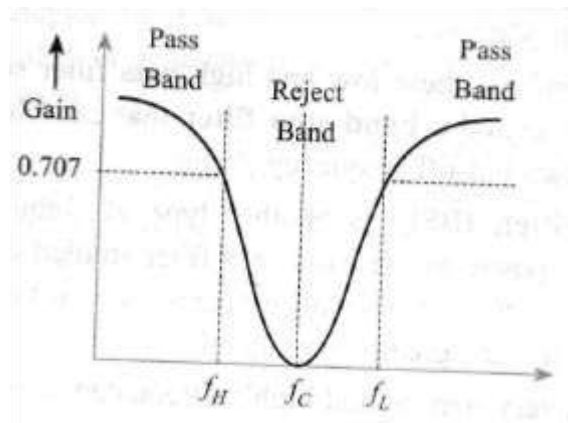
- BPFs are also used in optics like Lasers, LIDARS, etc. ○ BPFs are extensively used in wireless transmitters and receivers.
- BPFs are used in electronic devices like Sonar, Seismology; and medical applications like ECG, and electrocardiograms.
- BPFs are extensively used for Audio signal processing, where a particular range of frequencies of sound is required while removing the rest.

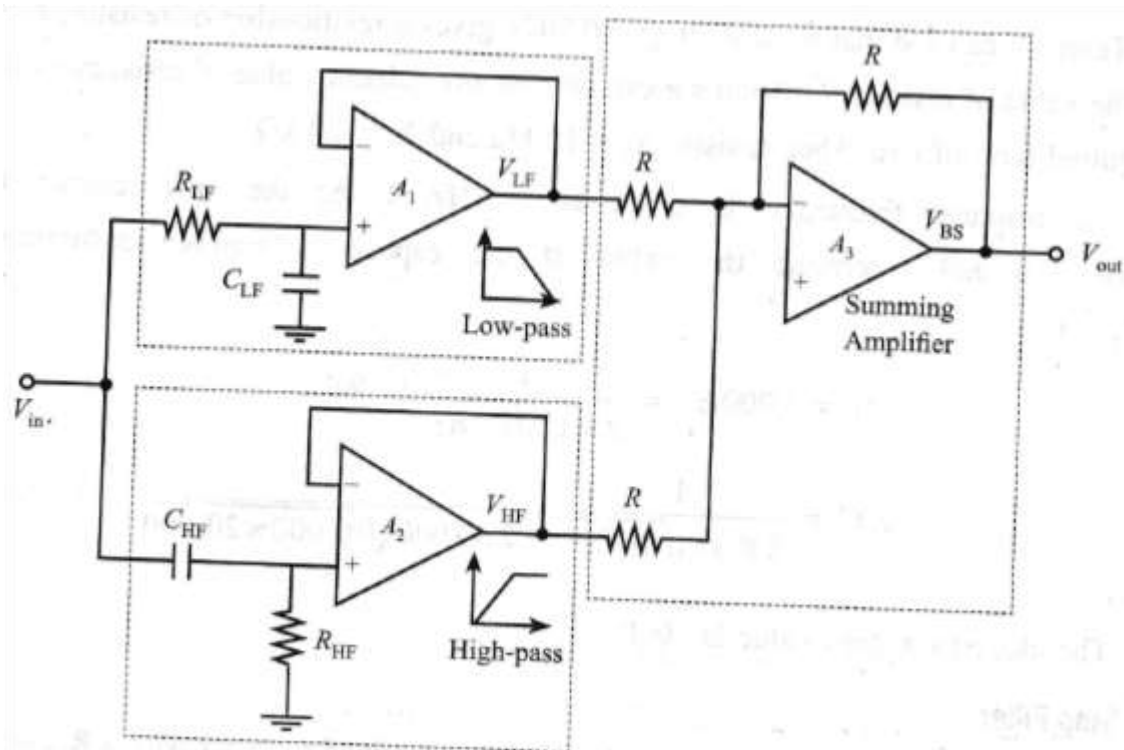
Band-Stop Filter (Band-Reject Filter):

Band-Stop Filter (BSF) is another type of frequency selective circuit that functions in exactly opposite to the band-pass filter. BSF passes all frequencies with the exception of those within a specified stop band, which are attenuated. If this stop band is very narrow and highly attenuated over a few hertz, then the band-stop filter is referred as a *notch filter*.

Wide Band-Reject Filter:

The frequency response curve and the circuit diagram of a wide band-reject filter is given below.





Example 2:

Design a basic wide-band, RC band stop filter with a lower cut-off frequency of 200Hz and a higher cut-off frequency of 800Hz. Find the geometric center frequency, -3dB bandwidth and Q of the circuit.

Given: $f_L = 200 \text{ Hz}$, $f_H = 800 \text{ Hz}$

We have

$$f = \frac{1}{2\pi RC} \text{ Hz}$$

The upper and lower cut-off frequency points for a band stop filter can be found using the same formula as that for both the low and high pass filters.

Assuming a capacitor, C value for both filter sections of $0.1 \mu\text{F}$, the values of the two frequency determining resistors, R_L and R_H are calculated as follows.

High Pass Filter Section

$$f_L = \frac{1}{2\pi R_L C} = 200 \text{ Hz} \text{ and } C = 0.1 \mu\text{F}$$

$$\therefore R_L = \frac{1}{2\pi \times 200 \times 0.1 \times 10^{-6}} = 7958 \Omega \text{ or } 8 \text{ k}\Omega$$

Low Pass Filter Section

$$f_H = \frac{1}{2\pi R_H C} = 800 \text{ Hz} \text{ and } C = 0.1 \mu\text{F}$$

$$\therefore R_H = \frac{1}{2\pi \times 800 \times 0.1 \times 10^{-6}} = 1990 \Omega \text{ or } 2 \text{ k}\Omega$$

From this, we can calculate the geometric center frequency, f_C as:

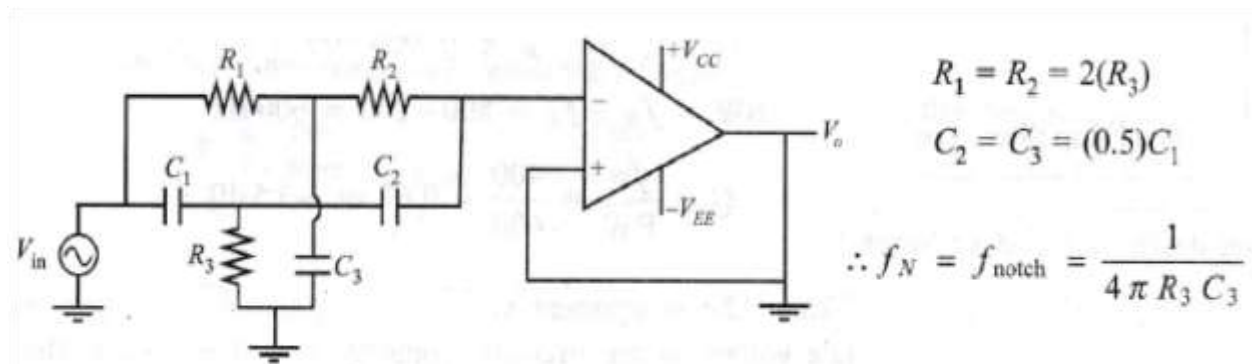
$$f_C = \sqrt{f_L \times f_H} = \sqrt{200 \times 800} = 400 \text{ Hz}$$

$$\text{BW} = f_H - f_L = 800 - 200 = 600 \text{ Hz}$$

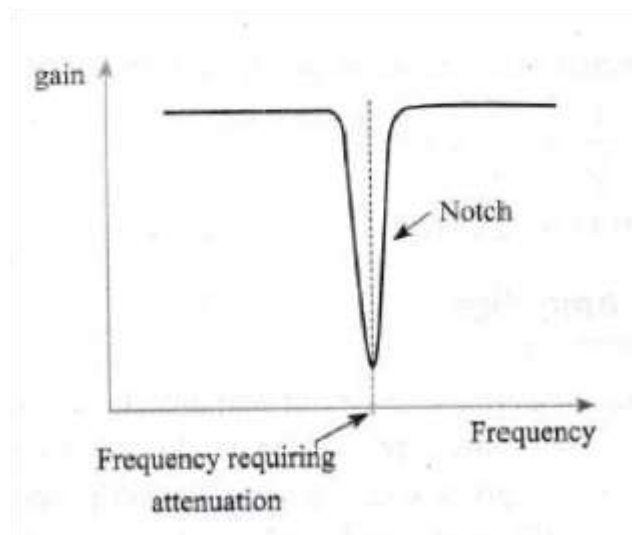
$$Q = \frac{f_C}{\text{BW}} = \frac{400}{600} = 0.67 \text{ or } -3.5 \text{ dB}$$

Narrow Band-Stop Filter (Notch Filter):

Notch filters are highly selective, high- Q form of band-stop filter, which can be used to reject a single or very small band of frequencies. The most common notch filter design is the twin-T notch filter network (shown below).



A twin-T network offers very high reactance at the resonance frequency and very low reactance at frequency off-resonance. In the circuit diagram, very low frequency signals find their way to the output via low-pass filter (formed by $R_1 - R_2 - C_3$); and very high frequency signals find their way to the output via high-pass filter (formed by $C_1 - C_2 - R_3$). Hence, in an intermediate band of frequencies, both filters pass signals to the output; due to cancellation of +ve phase shift of high-pass filter with the -ve phase shift of the low-pass filter.



Example 1:

Design a two op-amp narrow-band, RC notch filter with a center notch frequency, f_N of 1 kHz and a -3dB bandwidth of 100 Hz. Use 0.1 μ F capacitors in your design and calculate the expected notch depth in decibels.

Given: $f_N = 1000$ Hz, BW = 100 Hz and $C = 0.1$ μ F.

1. Calculate value of R for the given capacitance of 0.1 μ F

$$R = \frac{1}{4\pi f_N C} = \frac{1}{4\pi \times 1000 \times 0.1 \times 10^{-6}}$$

$$\therefore R = 795 \Omega \text{ or } 800 \Omega$$

2. Calculate value of Q

$$Q = \frac{f_N}{BW} = \frac{1000}{100} = 10$$

3. Calculate value of feedback fraction k

$$K = 1 - \frac{1}{4Q} = 1 - \frac{1}{4 \times 10} = 0.975$$

4. Calculate the values of resistors R_3 and R_4

$$K = 0.975 = \frac{R_4}{R_3 + R_4}$$

Assume $R_4 = 10$ k Ω , then R_3 equals:

$$R_3 = R_4 - 975 R_4 = 10000 - 0.975 \times 10000,$$

$$\therefore R_3 = 250 \Omega$$

5. Calculate expected notch depth

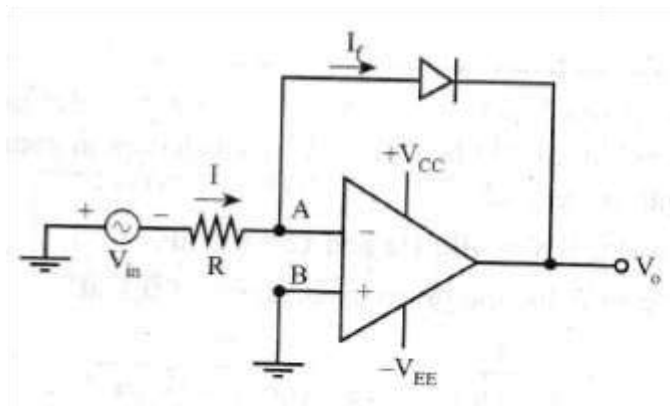
in decibels, (dB) $\frac{1}{Q} = \frac{1}{10} = 0.1$

$$f_{N(dB)} = 20 \log (0.1) = -20 \text{ dB}$$

NON LINEAR AMPLIFIER:

Non linearity is the behavior of a circuit, particularly an amplifier, in which the output signal strength do not vary in direct proportion to the input signal strength. A non-linear amplifier is a circuit which gives non linear relationship between its input and output signals.

The Non linear amplification can be achieved in a simple way by just connecting a non-linear device such as PN-junction diode in the feedback path. In the circuit shown is the following Figure, large change in input voltage causes small change in the output voltage. This circuit is a log amplifier, hence the output voltage is logarithm of the input voltage.



The above Figure shows a non-linear amplifier, where diode 'D' is used in negative feedback path. By virtual ground concept; as node B is grounded, node A will be virtually grounded. Therefore, $V_A = 0$.

since $V_A = 0$.

Let I_f be the current through the diode. The voltage across diode is $V_A - V_0$. Since, $V_A = 0$, the voltage across diode is $-V_0$.

$$\text{Diode equation:} \quad -V_0 = \eta V_T \ln \left[\frac{I_f}{I_r} \right]$$

Where, V_T – Voltage equivalent of Temperature

I_f – Diode forward current I_r –

Diode reverse saturation current.

Since, current through the Op-Amp is negligible; $I = I_f$

$$\text{Therefore,} \quad I_f = I = \frac{V_{in}}{R}$$

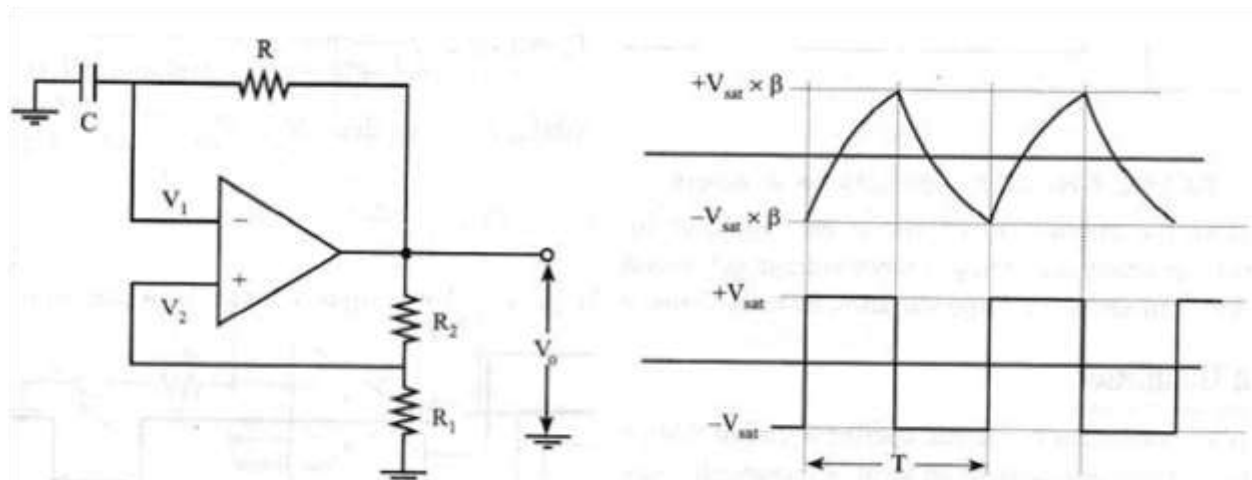
$$\text{Gives, } V_0 = \eta V_T \ln \left[\frac{V_{in}}{I_r R} \right] = \eta V_T \ln \left[\frac{V_{in}}{V_{ref}} \right] \quad I_r R = V_{ref} \text{ is a constant.}$$

The above equation shows that, the output voltage is a logarithmic function of input voltage.

Applications: Non-linear Amplifiers are used in AC bridge balance detectors.

RELAXATION OSCILLATOR:

Relaxation oscillator is a non-linear electronic oscillator circuit that generates a continuous non-sinusoidal output signal in the form of rectangular wave, triangular wave or a saw-tooth wave. The time period of non-sinusoidal output depends on the charging time of the capacitor connected in the oscillator circuit. The relaxation oscillator basically contains a feedback loop that has a switching device in the form of transistor, relays, operational amplifiers, comparators, or a tunnel diode that charges a capacitor through a resistance till it reaches a threshold level then discharges it again. The following Figure shows the basic circuit of an Op-Amp based relaxation oscillator.



Assume that, the output is initially in positive saturation. As a result, voltage at non-inverting input of OpAmp is $+V_{SAT} * R_1 / (R_1 + R_2)$. This force the output to stay in positive saturation as the capacitor C is initially in fully discharged state. Capacitor C starts charging towards $+V_{SAT}$ through R. The moment the capacitor voltage exceeds the voltage appearing at the non-inverting input, the output switches to $-V_{SAT}$.

Now, the voltage appearing at the non-inverting input changes to $-V_{SAT} * R_1 / (R_1 + R_2)$. The capacitor starts discharging and after reaching zero, it begins to discharge towards $-V_{SAT}$. Again, as soon as

it becomes more negative than the voltage appearing at the non-inverting input of the Op-Amp, the output switches back to $+V_{SAT}$.

The expression for the time period of the output rectangular waveform is given by; $T = 2 RC \ln \left(\frac{1+\beta}{1-\beta} \right)$

In the above equation; the natural logarithm is used, which is logarithm to base e . By varying the value of resistor R , the time period of the output waveform can be varied.

β is the feedback fraction/ factor and is given by $\beta = R1 / (R1 + R2)$

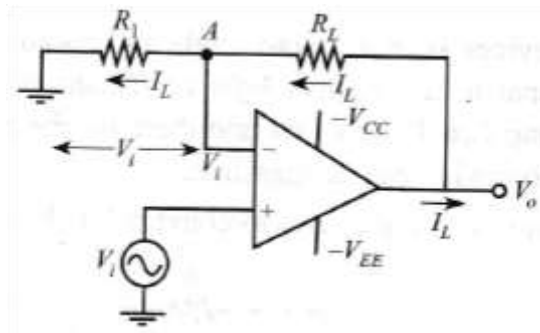
When the output voltage V_o is at $+V_{SAT}$ the feedback voltage is known as upper threshold voltage V_{UTP} and is given by $+V_{SAT} \times R1 / (R1 + R2)$

When the output voltage V_o is at $-V_{SAT}$ the feedback voltage is known as lower threshold voltage V_{LTP} and is given by $-V_{SAT} \times R1 / (R1 + R2)$

VOLTAGE TO CURRENT (V TO I) CONVERTER:

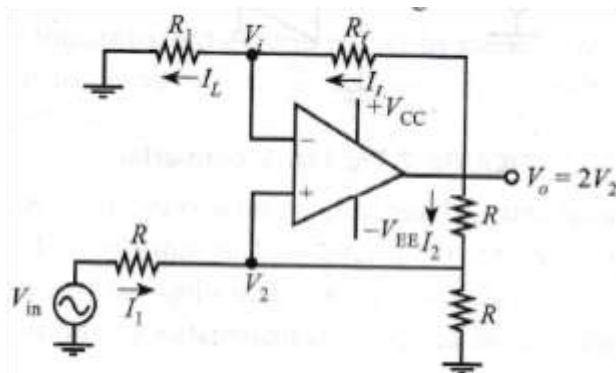
In many applications, we have to convert a voltage to a proportionate current. These voltage-to-current converter circuits can be of *two types*:

- a) **Voltage to Current Converter with Floating Load:** The circuit is shown in the following Figure, where R_L is the floating load.



Since, voltage at node A is V_i ; $V_{Vi} = I_{LL} R R_1$ Or, $I_L = \frac{V_i}{R_1}$
i.e., input voltage V_i is converted into an output current.

- b) **Voltage to Current Converter with Grounded Load:** The circuit is shown in the following Figure.



$$I_1 = \frac{V_{in} - V_2}{R} \quad I_2 = \frac{V_0 - V_2}{R}$$

The load current is given by; $I_L = I_1 + I_2$

$$\text{Therefore, } I_L = \frac{V_{in} - V_2}{R} + \frac{V_0 - V_2}{R} = \frac{V_{in} + V_0 - 2V_2}{R}$$

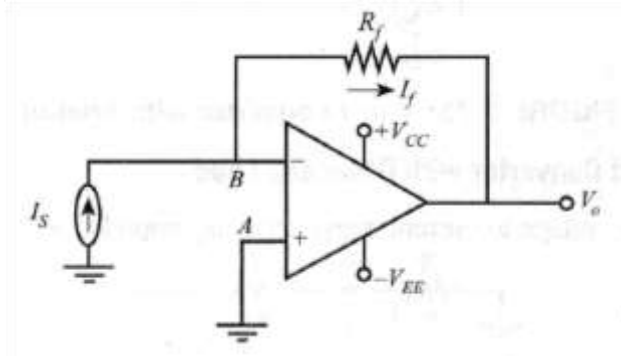
For a non-inverting amplifier, we know that; $A_v = 1 + \frac{R}{R} = 2$ So, $V_0 = 2V_2$

$$\text{Therefore, } I_L = \frac{V_{in} + 2V_2 - 2V_2}{R} \quad \text{Or, } I_L = \frac{V_{in}}{R}$$

Thus, the current I_L is proportionate to voltage.

CURRENT TO VOLTAGE (C TO V) CONVERTER:

Consider the simple Op-Amp circuit to convert I to V , as shown in the following Figure.



Since, current through the Op-Amp is negligible; $I_s = I_f$

$$I_s = I_f = \frac{V_B - V_0}{R_f}$$

By virtual ground concept; as node A is grounded, node B will be virtually grounded. Therefore, $V_B = 0$.

Therefore,

$$I_s = \frac{-V_0}{R_f} \quad \text{Or, } V_0 = I_s R$$

Thus, output is proportional to the input current I_s , and the circuit works as I to V converter.

VOLTAGE REGULATORS

All electronic systems that we use daily, requires a stable power supply voltage source; and voltage regulators accomplish that. *Voltage regulator* is a circuit that keeps the output voltage constant under all operating conditions. *Voltage regulation* is the process of keeping a voltage steady under conditions of changing applied voltage, changing load and temperature. There are *two types* of voltage regulators: *shunt* and *series*.

Need for Regulators:

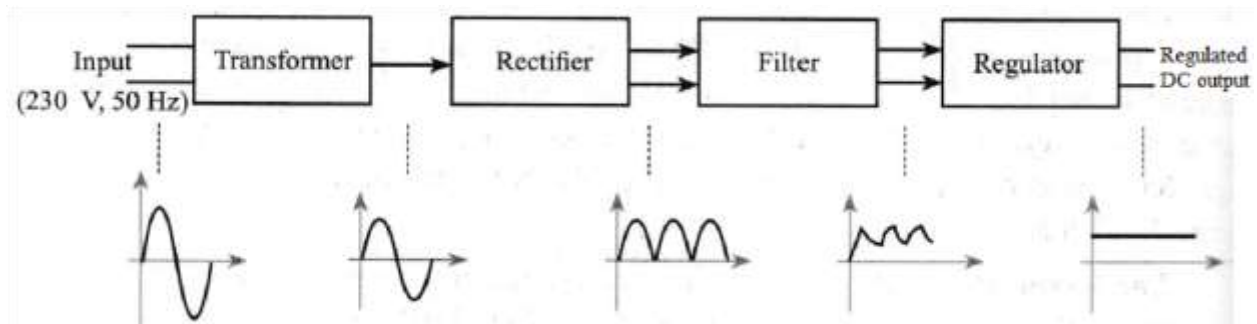
In ordinary power supplies, the voltage regulation is very poor. The DC output voltage changes appreciably with change in load current. The output voltage also changes due to fluctuations in the input AC supply.

This is due the following reasons:

1. In practice, there are considerable fluctuations in line voltage caused by external factors. This changes the DC output voltage. Most of the electronic circuits will refuse to work satisfactorily on such output voltage fluctuations. Hence, regulated power supply is the solution.

2. The internal resistance of ordinary power supply is relatively large ($> 30 \Omega$). Therefore, output voltage is affected by the amount of load current drawn from the supply. These variations in DC voltage may cause erratic operation of circuits. Without stable potentials, circuit performance degrades and if the variations are large enough, the components may get destroyed. In order to avoid this, regulated power supply is used.

Input to the voltage regulator is unregulated pulsating DC obtained from filter rectifier. Its output is constant DC voltage which is almost ripple free. The following Figure shows block diagram of regulated power supply.



The *transformer* provides voltage transformation and electrical isolation between the input power supply (AC mains) and the DC output. The *rectifier circuit* changes the AC voltage appearing across the transformer secondary to DC (unidirectional output). The rectifier circuit always has some AC content known as ripple. The *filter circuit* smoothens the ripple of the rectifier circuit. The *regulator* is a type of feedback circuit that ensures that the output DC voltage does not change from its nominal value due to change in line voltage or load current.

Factors Affecting the Load Voltage:

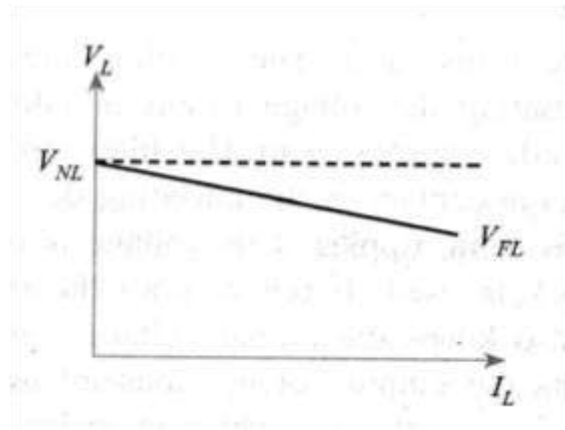
The variables affecting the load voltage in a power supply are given below:

1. **Load current (I_L):** Ideally the output voltage should remain constant in spite of changes in the load current, but practically the power supply without regulator, the load voltage decreases as load current, I_L , increases. For practical power supply regulator, the load voltage must be constant through load to full load condition.
2. **Line voltage:** The input to the rectifier is AC (230 V) is the line voltage. This input decides the output voltage level. If input changes, output also changes. So this affects the performance of power supply. So ideally voltage must remain constant irrespective of any changes in the line voltage.
3. **Temperature:** In the power supply, the rectifier unit is used which uses PN-junction diode. As the diode characteristics are temperature dependent, the overall performance of the power supply is temperature dependent.

Performance Parameters of a Power Supply:

The power supply is judged by some parameters, called as *performance parameters*. These performance parameters are explained below:

1. **Line Regulation:** If the input to the rectifier unit i.e. 230 V changes, the output DC of rectifier will also change and since the output of rectifier is applied to the regulator, the output of regulator will also vary. Thus the source causes the change in output. This is as *source regulation* or *line regulation*. It is defined as the change in regulated DC output for a given change in input (line) voltage. Ideally the source regulation should be zero and practically it should be as low as possible.
2. **Load Regulation:** *Load regulation* is defined as the change in the regulated output voltage when load current is changed from zero (no load) to maximum value (full load). The load regulation ideally should be zero, but practically it should be as small as possible. The following Figure shows the load regulation characteristics. Percentage load regulation = $\left[\frac{V_{NL} - V_{FL}}{V_{FL}} \right] * 100$



3. **Voltage Stability factor (S_V):** Voltage stability factor shows the dependency of output voltage on the input line voltage. Voltage stability factor is defined as the percentage change in the output voltage which occurs per volt change in input voltage, where load current and temperature are assumed to be constant. Smaller the value of this factor, better is the performance of power supply.
4. **Temperature Stability Factor (S_T):** As in the chain of power supply we are using semiconductor devices (diodes in rectifier block) the output voltage is temperature dependent. Thus the temperature stability of the power supply will be determined by temperature coefficients of various temperature sensitive semiconductor devices. So, it is better to choose the low temperature coefficient devices to keep output voltage constant and independent of temperature. S_T must be as small as possible, and ideally it should be zero for a power supply.
5. **Ripple Rejection Factor (RR):** The output of rectifier and filter consists of ripples. Ripple rejection is defined as a factor which shows how effectively the regulator rejects the ripples and attenuates it from input to output. As ripples in the output are small compared to input, the RR is very small and in dB, it is in negative value.
$$\text{Ripple rejection factor} = \frac{V_{\text{RIPPLE (OUTPUT)}}}{V_{\text{RIPPLE (INPUT)}}}$$

When expressed in decibels, ripple rejection equals $20 \log \left[\frac{V_{\text{RIPPLE (OUTPUT)}}}{V_{\text{RIPPLE (INPUT)}}} \right] \text{ dB}$

$$\text{Also, } V_{\text{RIPPLE (output)}} = \frac{V_{\text{RIPPLE (INPUT)}}}{1 + \text{Loop Gain}}$$

Example: Two power supplies A and B are available in the market. Power supply A has no-load and full-load voltages of 40 V and 30 V respectively; whereas these values are 30 V and 28 V for power supply B. Which one do you think is better power supply?

Supply A: $V_{NL} = 40 \text{ V}$, $V_{FL} = 30 \text{ V}$

$$\% \text{ Voltage regulation} = [(V_{NL} - V_{FL})/V_{FL}] * 100 = [(40 - 30)/30] * 100 = 33 \%$$

Supply B: $V_{NL} = 30 \text{ V}$, $V_{FL} = 28 \text{ V}$

$$\% \text{ Voltage regulation} = [(V_{NL} - V_{FL})/V_{FL}] * 100 = [(30 - 28)/28] * 100 = 7 \%$$

Conclusion: The power supply which has lower voltage regulation is better. Hence, power supply B is better than power supply A.

Example: A regulated power supply operates from $220 \pm 20 \text{ VAC}$. It produces a no-load regulated output voltage of $24 \pm 0.5 \text{ VDC}$. Also, the regulated output voltage falls from 24 VDC to 23.8 VDC as the load changes from no-load to full-load condition for the nominal value of input voltage. Determine (a) line regulation and (b) load regulation.

$$\text{Line regulation} = (24.5 - 23.5)/24 = 1/24 = 0.0417 = 4.17\%$$

$$\text{Load regulation} = (24 - 23.8)/23.8 = 0.2/23.8 = 0.0084 = 0.84\%$$

Example: A regulated power supply provides a ripple rejection of -80dB . If the ripple voltage in the unregulated input were 2V , determine the output ripple.

Ripple rejection in dB is given by;

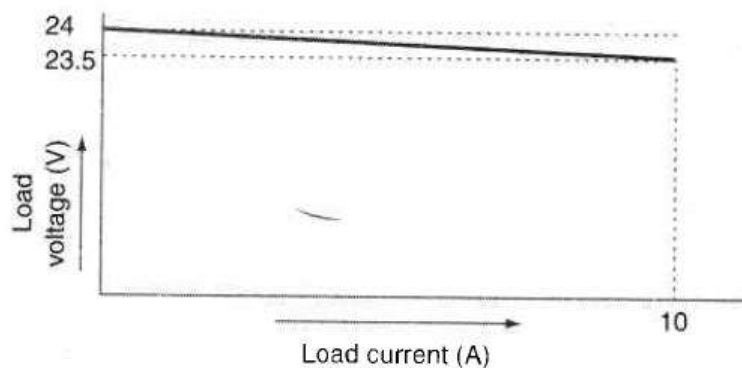
$$20 \log \left[\frac{V_{\text{RIPPLE (OUTPUT)}}}{V_{\text{RIPPLE (INPUT)}}} \right] \text{dB} = -80\text{dB}$$

Or $\log \left[\frac{V_{\text{RIPPLE (OUTPUT)}}}{V_{\text{RIPPLE (INPUT)}}} \right] = -4$

Or $\left[\frac{V_{\text{RIPPLE (OUTPUT)}}}{V_{\text{RIPPLE (INPUT)}}} \right] = 10^{-4}$

Therefore, output ripple = $2 * 10^{-4} \text{V} = 0.2\text{mV}$

Example: The following Figure shows load voltage versus load current characteristics of a regulated power supply. Determine the output impedance of the power supply.



Output impedance is given by ratio of change in the output voltage for known change in the load current. From the given characteristic curve, output impedance = $(24 - 23.5)/(10 - 0) = 0.5/10 = 0.05\Omega = 50\text{m}\Omega$.

Three-Terminal Regulators:

Three-terminal regulators require no external components. These are available in fixed output voltage (positive and negative) as well as adjustable output voltage (positive and negative) types with current rating 100mA , 500mA , 1.5A and 3.0A .

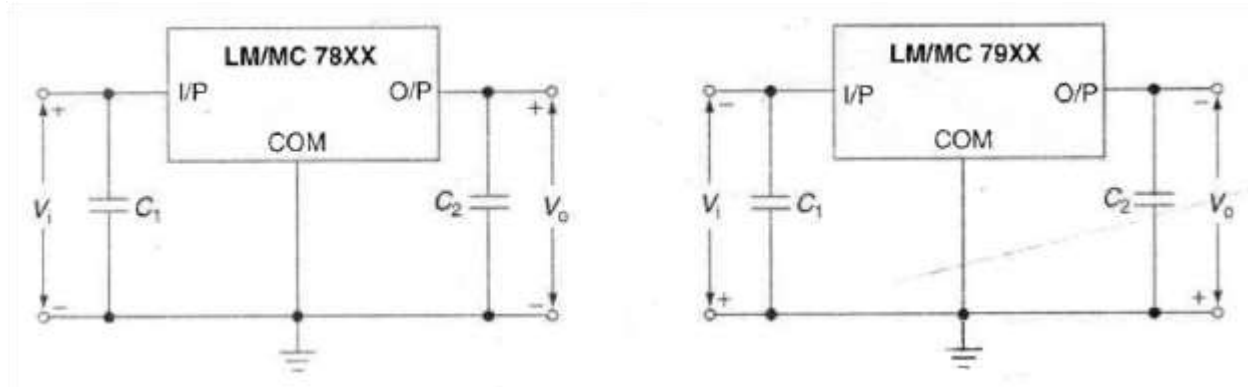
- LM/MC 78XX-series and LM 140XX/340XX-series are the popular three-terminal positive output voltage regulators.
- LM/MC 79XX-series and LM 120XX/320XX-series are the popular three-terminal negative output voltage regulators.
- LM 117/217/317 is common adjustable positive output voltage regulators.
- LM 137/237/337 is common adjustable negative output voltage regulators.

✓ A two-digit number in place of “XX” indicates the regulated output voltage.

The minimum unregulated input to regulated output differential voltage required for the regulator to produce the intended regulated output voltage is known as *dropout voltage*. For example, consider a 5V regulator with a 2V dropout voltage; for this to give a regulated output, the input voltage must be least equal to the

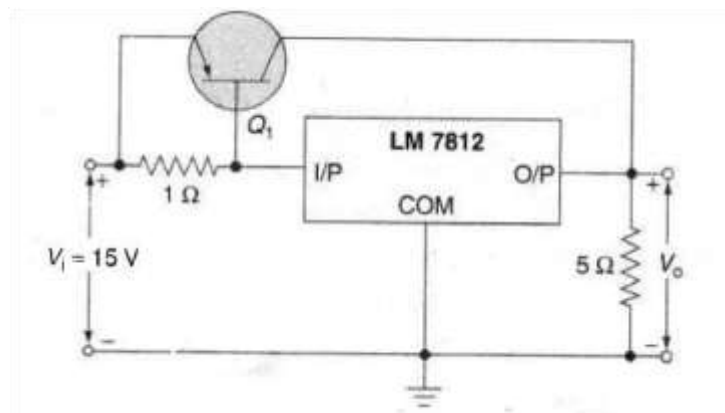
output voltage (5V) plus the dropout voltage (2V), which is 7V; any input below 7V will result into unregulated voltage output.

The following Figures show the basic application circuits using LM/MC 78XX-series and Lm/MC 79XXseries three-terminal regulators.



Basic Application Circuits using Three-Terminal Regulators

Example: Refer three-terminal regulator circuit of following Figure. Determine (a) load current; (b) current through LM 7812; (c) current through external transistor; (d) power dissipated in LM 7812. Take $V_{BE}(Q_1) = 0.7V$.



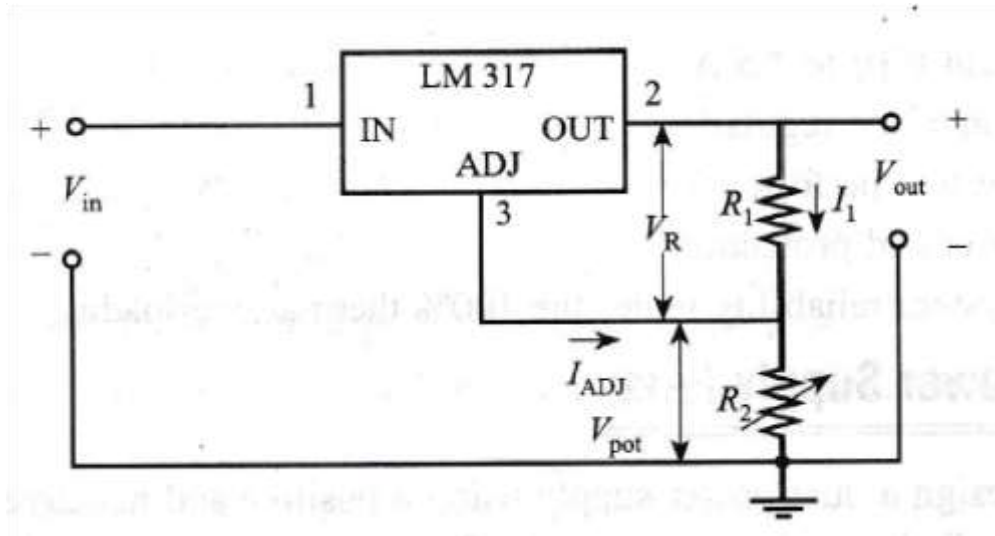
- (a) Load current = $12/5 = 2.4A$
 (b) Current through regulator = $0.7/1 = 0.7A$
 (c) Current through external transistor = $2.4 - 0.7 = 1.7A$
 (d) Voltage appearing at regulator input = $15 - 0.7 = 14.3V$ (e) Power dissipated in the regulator = $(14.3 - 12) * 0.7 = 1.61W$.

Adjustable Voltage Regulator:

An adjustable voltage regulator is a kind of regulator, whose regulated output voltage can be varied over a range. There are positive adjustable voltage regulators and negative adjustable regulators in practice.

LM317 is a classic example of positive adjustable voltage regulator, whose output voltage can be varied over a range of 1.2 V to 57 V. LM337 is an example of negative adjustable voltage regulator. LM337 is

actually a compliment of LM317 which are similar in operation and design with the only difference being polarity of regulated output voltage.



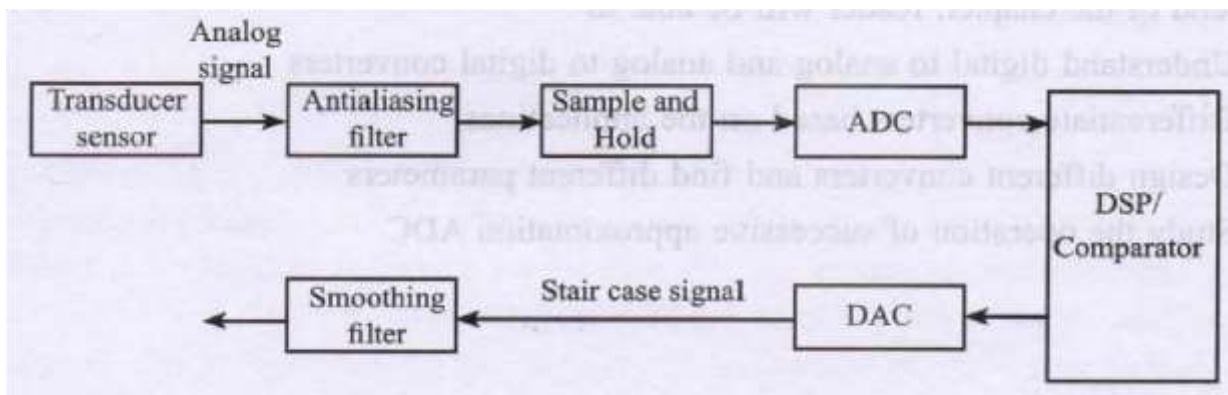
Connection of LM317 Adjustable Voltage Regulator

The resistors R_1 and R_2 determine the output voltage V_{out} . The resistor R_2 can be adjusted to get the output voltage in the range of 1.21 V to 57 V. The output voltage is given by;

$$V_{out} = V_R (1 + R_2/R_1) + I_{ADJ}R_2$$

D TO A & A TO D CONVERTERS

The digital system such as computers also need to communicate with physical processes and with people through analog signals. So there is a need of digital to analog converters.



Typical A/D and D/A Converter

Naturally available signal is analog in form, and may be obtained from *sensor or transducer*. This analog signal is band limited by *anti-aliasing filter*. The signal is then *sampled* at a frequency rate, more than twice the maximum frequency of the band limited signal. The sampled signal is held constant by *hold* circuit while conversion is taking place. The discrete signal from the sample and hold circuit is fed to analog to digital converter (ADC). The ADC gives digital output signal that can be easily processed, stored, or transmitted by digital systems or computer system.

The digital signal is converted back to by *digital to analog converter* (DAC). The output of DAC is usually stair-case waveform, which is passed through smoothing filter to reduce the quantization noise. The diagram shown in the above Figure can be used in the applications such as digital signal processing, digital audio mixing, music and video synthesis, data acquisition, pulse code modulation, and microprocessor instrumentation.

BASIC DAC TECHNIQUES:

The DAC converts digital or binary data into its equivalent analog value. The symbolic representation of an n-bit DAC is given below:



n-bit DAC

The DAC output can either be a voltage or current signal. For a voltage output DAC, the conversion characteristic can be expressed by;

$$V_0 = k V_{FS} (b_1 2^{-1} + b_2 2^{-2} + b_3 2^{-3} + \dots + b_n 2^{-n})$$

Where, V_0 – Output voltage

V_{FS} – Full scale output voltage k – Scaling factor (usually 1)

b_1, \dots, b_n – n-bit binary fractional word with decimal point located at the left

b_1 – MSB with a weight = $V_{FS}/2$ b_n – LSB with a weight = $V_{FS}/2^n$

Performance Parameters of DAC:

1. **Resolution:** Resolution is the number of various analog output values that is provided by a DAC.

For n-bit DAC; $Resolution = 2^n$

Resolution can also be defined as the ratio of change in output voltage resulting from a change of LSB at the digital input. For n-bit DAC; $Resolution = V_{OFS}/2^n - 1$

Where, V_{OFS} – Full scale output voltage.

If we know the resolution, we can obtain input-output relation for DAC:

$$V_0 = Resolution \times b \text{ Where,}$$

b – Decimal values of digital input.

For example, in a 4-bit system, using ladder, the LSB has a weight of $\frac{1}{16}$. This means that, the smallest increment in the output voltage is $\frac{1}{16}$ of the input voltage. If we assume that, this 4-bit system has input voltage levels of +16 V; (since has a weight of $\frac{1}{16}$) a change in LSB results in a change of 1 V in the output. Thus, the output voltage changes in steps of 1 V.

Hence, this converter can be used to represent analog voltages from 0 to +15 V in 1-V increments. But, this converter cannot be used to resolve voltages into increments smaller than 1V. If we desire to produce +4.2 V, using this converter, the actual output voltage would be +4.0 V. This converter is not capable of distinguishing voltages finer than 1 V, which is the resolution of the converter.

If we want to represent voltages to a finer resolution, we would have to use a converter with more input bits. For example, the LSB of a 10-bit converter has a weight of $1/1024$. If this converter has a +10 V full-scale output, the resolution is approximately, $+10 * \frac{1}{1024} = 10 \text{ mV}$.

Problem: What is the resolution of a 9-bit D/A converter which uses a ladder network? What is the resolution expressed as a percentage? If the full-scale output voltage of this converter is +5 V, what is resolution in volts?

Solution: The LSB in a 9-bit D/A converter has a weight of $\frac{1}{512}$. Thus, this converter has a resolution of 1 part in 512.

The resolution expressed in percentage is $\frac{1}{512} * 100 \text{ percent} = 0.2 \%$.

The voltage resolution is obtained by multiplying the weight of the LSB by the full-scale output voltage. Thus, the resolution in volts is: $\frac{1}{512} * 5 = 10 \text{ mV}$.

Problem: How many bits are required at the input of a converter, if it is necessary to resolve voltage to 5 mV and the ladder has +10 V full-scale?

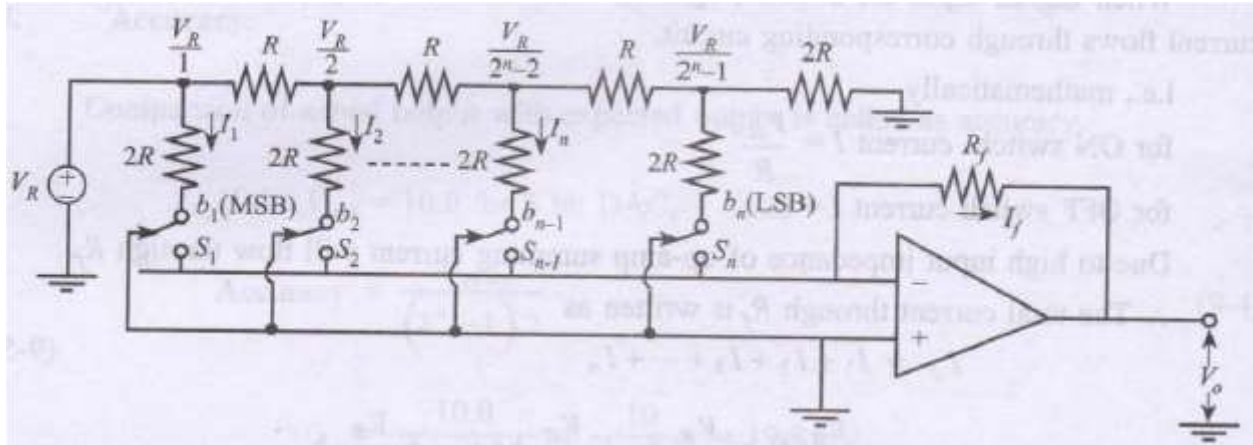
Solution: The LSB of an 11-bit D/A converter has a resolution of $\frac{1}{2048}$. This would provide a resolution at the output of $\frac{1}{2048} * 10 = +5 \text{ mV}$.

2. **Accuracy:** The accuracy of the D/A converter is primarily a function of the accuracy of the precision resistors used in the ladder and the precision of the reference voltage supply used.

Accuracy is a measure of how close the actual output voltage is to the theoretical output value.

R-2R Ladder type DAC:

In R-2R ladder, only two values, R and 2R are used. The circuit diagram is given below:



Each binary bit connects switch either to ground (non-inverting input) or to the inverting terminal of OpAmp. Due to virtual ground, both the positions of the switches are at ground potential, and currents through the resistances are constant.

The current flowing through each of 2R resistances;

$$I_1 = \frac{V_R}{2R} \quad I_2 = \frac{V_R/2}{2R} = \frac{V_R}{4R} \quad I_3 = \frac{V_R/4}{2R} = \frac{V_R}{8R} \quad I_n = \frac{V_R/(2^{n-1})}{2R}$$

$$\text{But, } V_0 = -I_{\text{total}} R_f = -R_f (I_1 + I_2 + \dots + I_n)$$

$$\text{i.e., } V_0 = -R_f \left[\frac{V_R}{2R} b_1 + \frac{V_R}{4R} b_2 + \dots + \frac{V_R}{2^n R} b_n \right]$$

$$\text{Or, } V_0 = -\frac{V_R}{R} R_f [b_1 2^{-1} + b_2 2^{-2} + \dots + b_n 2^{-n}]$$

$$\text{If } R_f = R; \quad V_0 = -V_R [b_1 2^{-1} + b_2 2^{-2} + \dots + b_n 2^{-n}]$$

Advantages: (1) As it requires only two types of resistors, fabrication and accurate value of R-2R can be designed.

(2) Node voltage remains constant, and hence, slow down effect can be avoided.

Example 1:

The digital input for a 4 bit DAC is $D = 0111$. Calculate its output voltage take $V_{\text{REF}} = 15 \text{ V}$.

$$\text{Resolution} = \frac{V_{\text{REF}}}{2^n - 1} = \frac{15}{2^4 - 1} = 1 \text{ V / LSB}$$

$$\therefore V_o = \text{Resolution} \times D$$

$$D = \text{Decimal values (0111)} = 7$$

$$V_o = \frac{1 \text{ V}}{\text{LSB}} \times 7 = 7 \text{ V}$$

$$\therefore \boxed{V_o = 7 \text{ V}}$$

Example 2:

A 8 bit DAC having resolution of 22mV/LSB. Calculate V_{oFS} and output if the input is $(10000000)_2$.

Given: resolution = 22 mV, Input = $(10000000)_2$

$$\text{Resolution} = \frac{V_{oFS}}{2^n - 1}$$

$$22\text{mV} = \frac{V_{oFS}}{2^8 - 1}$$

$$V_{oFS} = 5.6 \text{ V}$$

$$D = \text{equivalent of } (10000000)_2 = 128$$

$$V_o = 22 \times 10^{-3} \times 128 = 2.8 \text{ V.}$$

Example 3:

Calculate output voltage produced by DAC, when output range is between 0 and 10 V for input binary number.

a) 10 (2 bit DAC) b) 0011

a) From equation (9-7) we can write,

$$V_o = 10 \text{ V} \left(1 \times \frac{1}{2} + 0 \times \frac{1}{4} \right) = 5 \text{ V}$$

b) From equation (9-7) we can write,

$$V_o = 10 \text{ V} \left(0 \times \frac{1}{2} + 0 \times \frac{1}{4} + 1 \times \frac{1}{8} + 1 \times \frac{1}{16} \right) \\ = 1.875 \text{ volts.}$$

Example 4:

Calculate the values of the LSB and full scale output for 4 bit DAC for 0 to 10 V range.

We have,

$$\text{LSB} = \frac{1}{2^4} = \frac{1}{16}$$

For 10 V range,

$$\text{LSB} = \frac{10 \text{ V}}{16} = 625 \text{ mV}$$

and

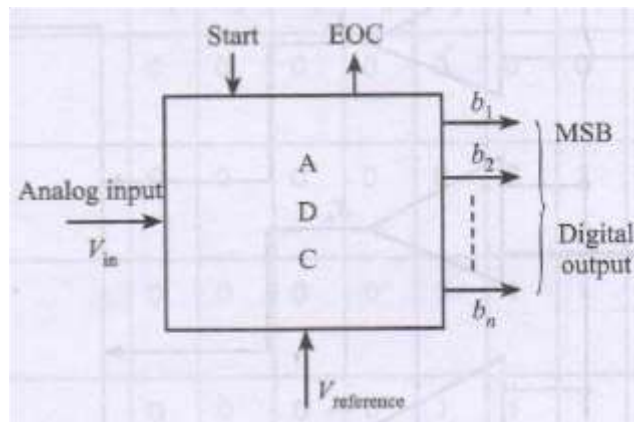
$$\text{MSB} = \left(\frac{1}{2} \right) \text{Fullscale} \\ = \frac{1}{2} \times 10 = 5 \text{ V}$$

Full scale output = Full scale voltage – 1 LSB

$$= 10 - 625 \text{ mV} = 9.375 \text{ V}$$

A-D CONVERTERS:

ADC takes the analog signal as input and converts into digital output. The functional diagram of DAC is given below:

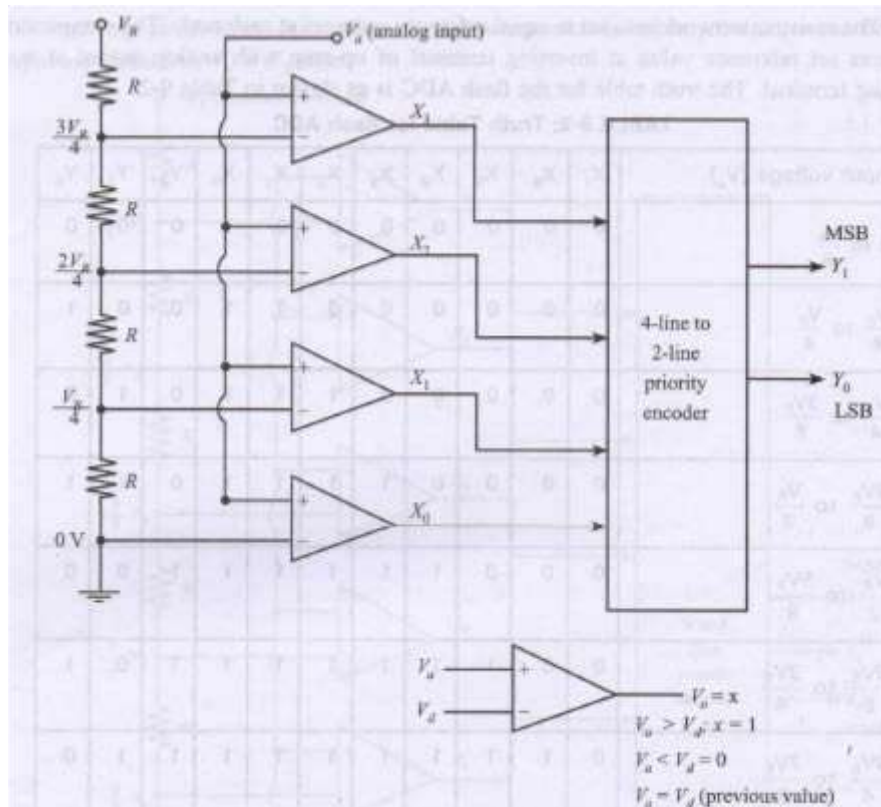


ADC is provided with two control inputs *start* (input to initiate the conversion) and *end of conversion* (output to indicate the end of conversion). *Direct type ADCs* and *Integrated type ADCs* are the two types of ADCs available.

Flash (Comparator/ Parallel) type ADC:

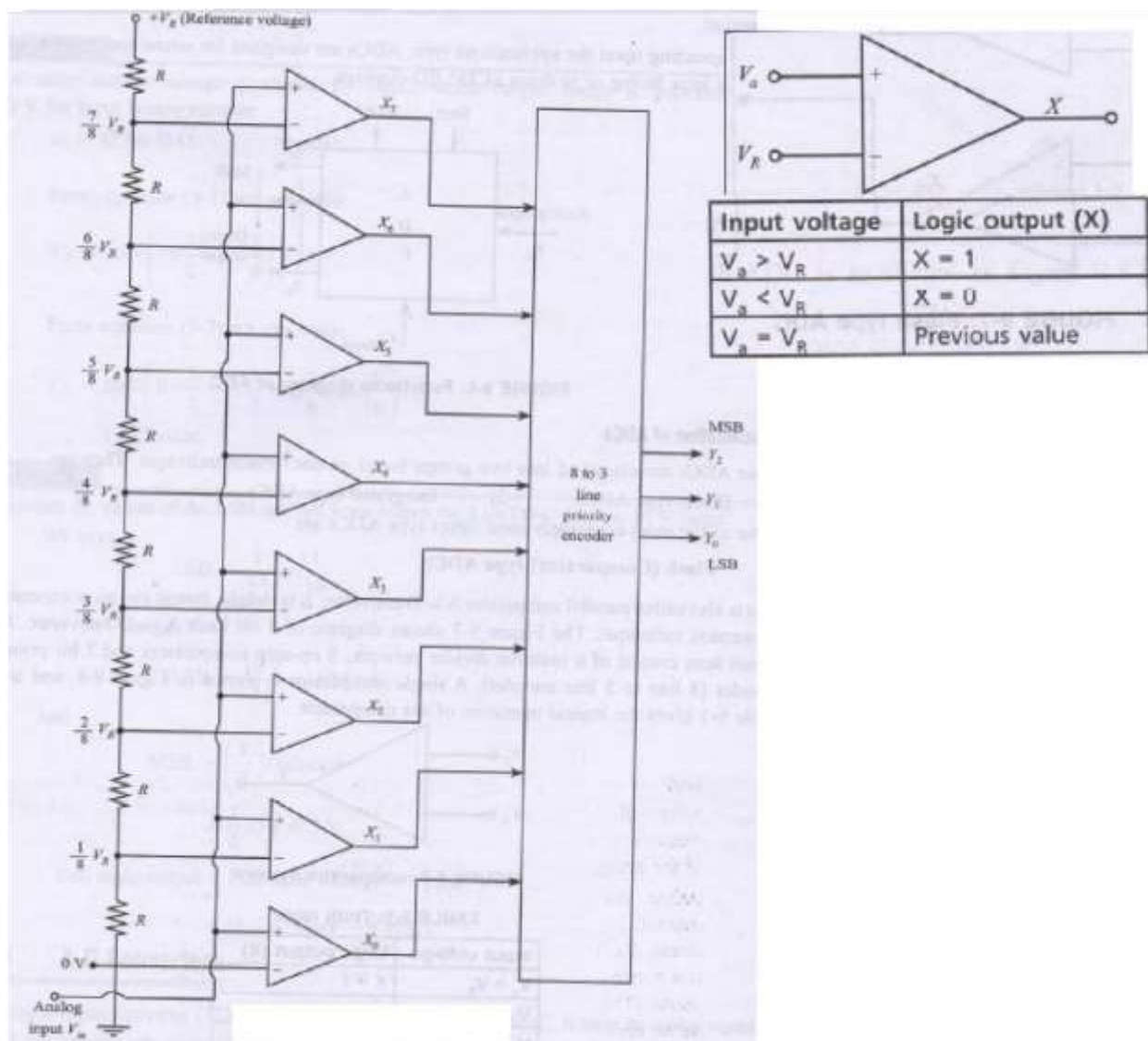
A simple, fast, but most expensive conversion technique.

A 2-bit Flash ADC:



Analog input voltage (V_a)	X_3	X_2	X_1	X_0	Y_1	Y_0
0 to $\frac{V_R}{4}$	0	0	0	1	0	0
$\frac{V_R}{4}$ to $\frac{2V_R}{4}$	0	0	1	1	0	1
$\frac{2V_R}{4}$ to $\frac{3V_R}{4}$	0	1	1	1	1	0
$\frac{3V_R}{4}$ to V_R	1	1	1	1	1	1

A 3-bit Flash ADC:



The resistive network is to set to equal reference voltages at each node. The comparactor compares set reference value at inverting terminal of Op-Amp with analog output at non-inverting terminal. The truthable for ADC is given below:

Input voltage (V_a)	X_7	X_6	X_5	X_4	X_3	X_2	X_1	X_0	Y_2	Y_1	Y_0
0 to $\frac{V_R}{8}$	0	0	0	0	0	0	0	1	0	0	0
$\frac{V_R}{8}$ to $\frac{V_R}{4}$	0	0	0	0	0	0	1	1	0	0	1
$\frac{V_R}{4}$ to $\frac{3V_R}{8}$	0	0	0	0	0	1	1	1	0	1	0
$\frac{3V_R}{8}$ to $\frac{V_R}{2}$	0	0	0	0	1	1	1	1	0	1	1
$\frac{V_R}{2}$ to $\frac{5V_R}{8}$	0	0	0	1	1	1	1	1	1	0	0
$\frac{5V_R}{8}$ to $\frac{3V_R}{4}$	0	0	1	1	1	1	1	1	1	0	1
$\frac{3V_R}{4}$ to $\frac{7V_R}{8}$	0	1	1	1	1	1	1	1	1	1	0
$\frac{7V_R}{8}$ to V_R	1	1	1	1	1	1	1	1	1	1	1

Advantages:

1. High speed

Disadvantages:

1. Number of comparators required is almost double for each added bit

Eg.: For 2-bit ADC; No. of Comparators = 4 (2^2)

For 3-bit ADC; No. of Comparators = 8 (2^3)

Successive Approximation type ADC:

The following Figure shows successive approximation ADC.

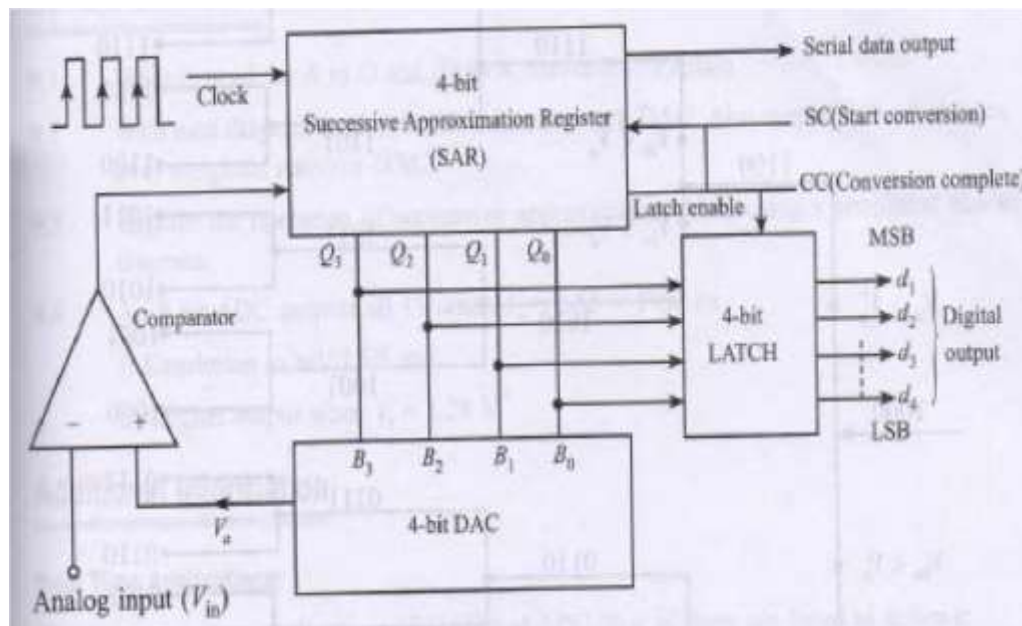
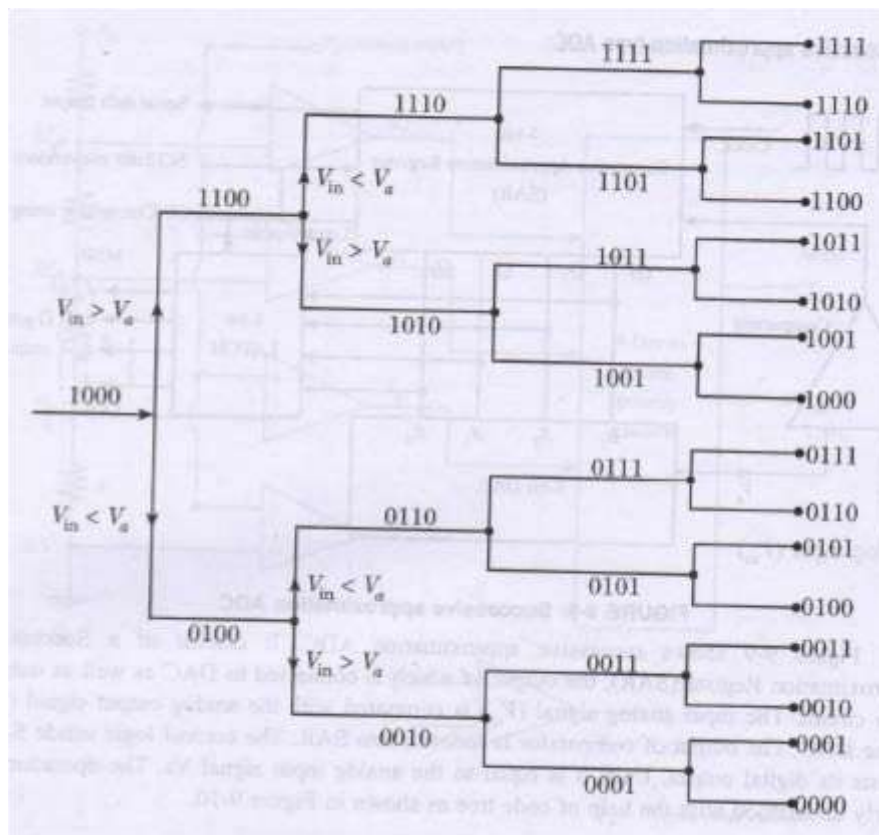


Figure shows a successive approximation register (SAR), the output of which is connected to DAC and output latch circuit. The input signal (V_{in}) is compared with the analog output signal (V_a) of the DAC. Output of the comparator is feedback into SAR. The control logic inside SAR adjusts its digital output; until it is equal to the analog input signal. The operation could be understood by the code tree given below.

At the start of conversion cycle, start conversion terminal is made high. On the first clock pulse, the output of the SAR is made 1000. The DAC produces an analog voltage (V_a) proportional to 1000. This analog voltage is compared with input analog signal (V_{in}).

If $V_{in} > V_a$, the comparator output will be high and SAR keeps Q3 high. On the other hand, if $V_{in} < V_a$, then the comparator output becomes low and SAR resets Q3 to low. If $V_{in} > V_a$, SAR follows the upward path in code tree and if $V_{in} < V_a$, SAR follows downward path.



The conversion time for n -bit successive approximation ADC is $(n + 2)$ clock periods.

Advantages:

1. Considerably good speed
2. Good resolution.

References

Books:

1. Charles H Roth and Larry L Kinney, Analog and Digital Electronics, Cengage Learning, 2019
2. Anil K Maini, Varsha Agarwal, Electronic Devices and Circuits, Wiley, 2012.

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https://www.tutorialspoint.com/linear_integrated_circuits_applications/index.htm

<https://www.elprocus.com/digital-to-analog-converter-dac-applications/>

<https://www.electronicshub.org/photodiode-working-characteristics-applications/>

<https://www.electrical4u.com/applications-of-op-amp/>

Question Bank:

Explain the working of construction and working of Astable multivibrator using 555 timer IC

Explain the working of Photodiode with necessary circuit diagram.

Explain the construction and working of Relaxation oscillator.

Explain the working of R2R DAC with necessary circuit diagram & derivations.

Explain the construction, working principles and characteristics of photodiodes.

Explain the construction, working principles and applications of light emitting diode (LED).

Explain peak detector circuit with a neat waveform.

With a neat diagram explain R-2R ladder network DAC.

Explain the operation of successive approximation ADC using simplified block diagram.

Explain peak detector circuit with a neat waveform.

Explain Astable multivibrator with neat circuit diagram and waveform.

University Questions:

- 1) a. Explain the construction, working and characteristics of photo diode. (06 Marks)
b. With hysteresis characteristics explain the working of Schmitt trigger circuit (Inverting). (06 Marks)
c. With a neat circuit diagram and mathematical analysis explain voltage divider bias circuit. (08 Marks)
- 2) a. Explain the working of R-2R ladder D to A converter. (06 Marks)
b. Explain successive approximation A to D converter. (06 Marks)
c. Show how IC-555 timer can be used as an astable multivibrator. (08 Marks)

- 3) a. With a neat diagram, explain the working principle of photocoupler. (08 Marks)
b. List the different types of BJT biasing. Derive the expression for collector emitter voltage (V_{CE}) for fixed bias circuit. (08 Marks)
c. Write a note on light emitting diode. (04 Marks)
- 4) a. Explain with neat diagram, the construction, working principle and characteristics equation of photodiode. (08 Marks)
b. With a neat waveform and circuit diagram, explain the working of monostable multivibrator. (06 Marks)
c. Explain with neat diagram R-2R ladder type DAC and derive the expression for V_0 . (06 Marks)

THE BASIC GATES

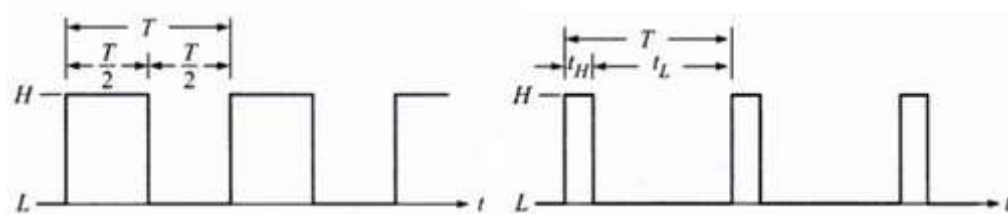
PREREQUISITES:

Electronic circuits and systems can be divided into two broad categories – *analog* and *digital*. Analog circuits are designed for use with small signals and are used in a linear fashion. Digital circuits are generally used with large signals and are considered nonlinear. Any quantity that changes with time can be represented as an analog signal or it can be treated as digital signal.

Digital electronics involves circuits that have exactly two possible states. A system having only two states is said to be *binary*. The binary number system is widely used in digital electronics.

Hexa-Decimal	Decimal	Binary	Hexa-Decimal	Decimal	Binary
0	0	0 0 0 0	8	8	1 0 0 0
1	1	0 0 0 1	9	9	1 0 0 1
2	2	0 0 1 0	A	10	1 0 1 0
3	3	0 0 1 1	B	11	1 0 1 1
4	4	0 1 0 0	C	12	1 1 0 0
5	5	0 1 0 1	D	13	1 1 0 1
6	6	0 1 1 0	E	14	1 1 1 0
7	7	0 1 1 1	F	15	1 1 1 1

The operation of electronic circuits can be described in terms of its voltage levels – *high* (H) level and *low* (L) level. This could be related to the binary number system by assigning L = 0 = F (false) and H = 1 = T (true).



Symmetrical Signal & Asymmetrical Signal

The frequency is defined as, $f = 1 / T$ where, T is the period of the signal.

Duty Cycle is a convenient measure of how symmetrical or how unsymmetrical a waveform is.

$$\text{Duty Cycle} = \frac{T_{on}}{T_{on} + T_{ooooo}}$$


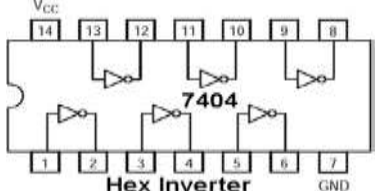
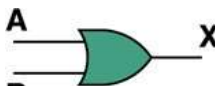
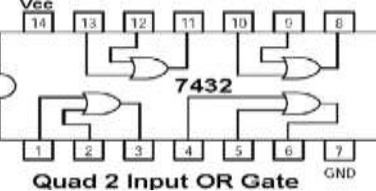
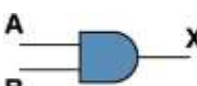
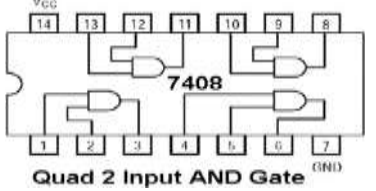
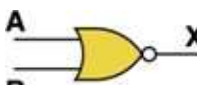
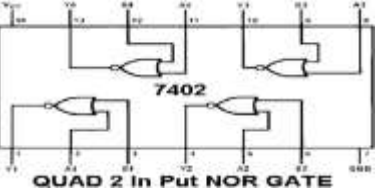
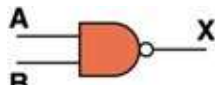
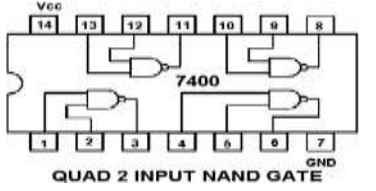
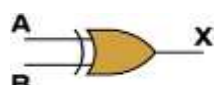
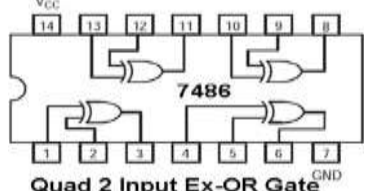
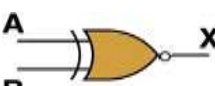
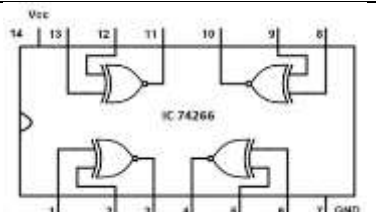
$$\text{Duty Cycle, H} = \frac{T_{on}}{T_{on} + T_{ooooo}}$$

$$\text{Duty Cycle, L} = \frac{T_{ooooo}}{T_{on} + T_{ooooo}}$$

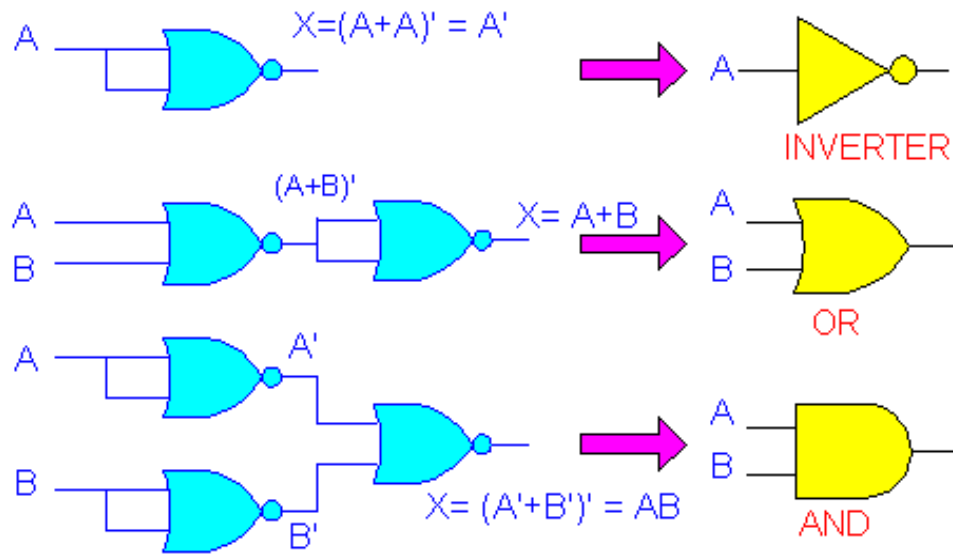


ANALOG AND DIGITAL ELECTRONICS

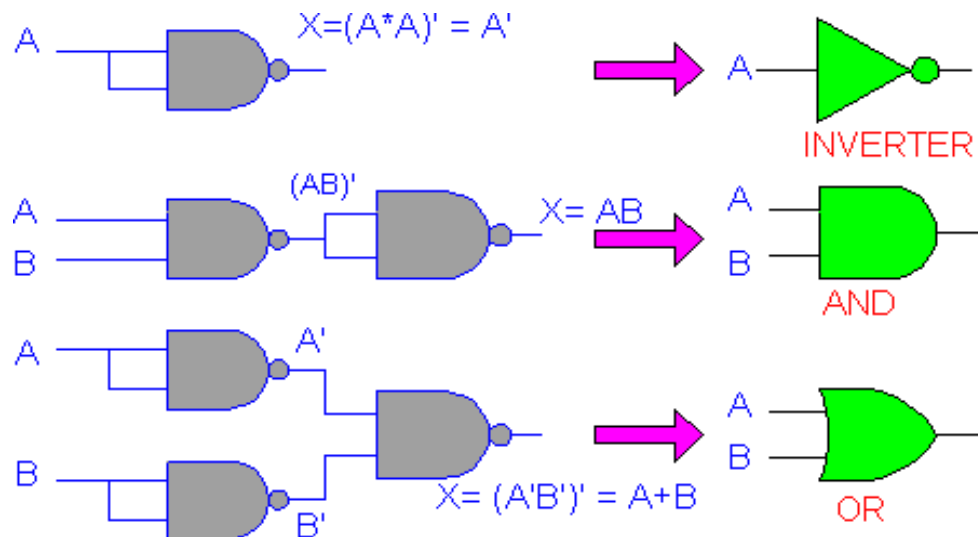
REVIEW OF LOGIC GATES:

Circuit Symbol	Truth Table			VHDL	IC Details
	A	B	X		
NOT Gate:  $XX = \bar{A}$	0	-	1	$X = \sim A$ $X \leq \text{not } A;$	 Hex Inverter
	1	-	0		
OR Gate:  $XX = A + B$	0	0	0	$X = A \mid B$ $X \leq A \text{ or } B;$	 Quad 2 Input OR Gate
	0	1	1		
	1	0	1		
	1	1	1		
AND Gate:  $XX = A \cdot B$	0	0	0	$X = A \& B$ $X \leq A \text{ and } B;$	 Quad 2 Input AND Gate
	0	1	0		
	1	0	0		
	1	1	1		
NOR Gate:  $XX = \overline{A + B}$	0	0	1	$X = \sim(A \mid B)$ $X \leq A \text{ nor } B;$	 QUAD 2 In Put NOR GATE
	0	1	0		
	1	0	0		
	1	1	0		
NAND Gate:  $XX = \overline{A \cdot B}$	0	0	1	$X = \sim(A \& B)$ $X \leq A \text{ nand } B;$	 QUAD 2 INPUT NAND GATE
	0	1	1		
	1	0	1		
	1	1	0		
XOR Gate:  $XX = A \oplus B$ $= \bar{A}B + A\bar{B}$	0	0	0	$X = A \wedge B$ $X \leq A \text{ xor } B;$	 Quad 2 Input Ex-OR Gate
	0	1	1		
	1	0	1		
	1	1	0		
XNOR Gate:  $XX = A \odot B$ $= \bar{A}B + AB$	0	0	1	$X = \sim(A \wedge B)$ $X \leq A \text{ xnor } B;$	 IC 74266
	0	1	0		
	1	0	0		
	1	1	1		

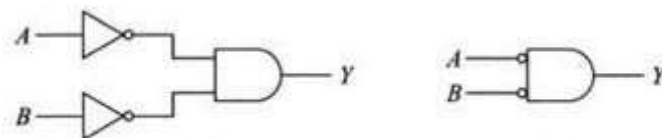
Universality of NOR Gate:



Universality of NAND Gate:



Bubbled AND Gate:



Bubbled AND gate and NOR gate are equivalent

De Morgan's First Theorem:

The complement of a sum equals the product of the complements.

$$\overline{A + B} = \bar{A} \cdot \bar{B}$$

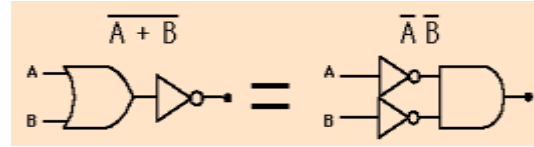
ANALOG AND DIGITAL ELECTRONICS

Proof:

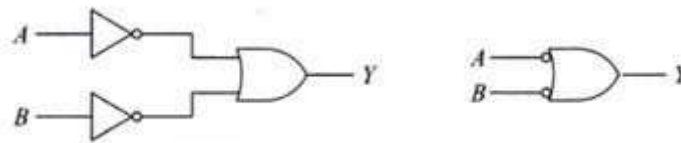
A	B	A+B	1 0 0 1	A	1 0 1 0	A.B
0	0	0	1	1	1	1
0	1	1	0	1	0	0
1	0	1	0	0	1	0
1	1	1	0	0	0	0

NOR Gate

Bubbled AND Gate



Bubbled OR Gate:



Bubbled OR gate and NAND gate are equivalent

De Morgan's Second Theorem:

The complement of a sum equals the product of the complements.

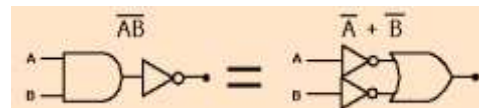
$$\overline{A+B} = \bar{A} \cdot \bar{B}$$

Proof:

A	B	AB	1 0 0 1	A	1 0 1 0	A + B
0	0	0	1	1	1	1
0	1	0	1	1	0	1
1	0	0	1	0	1	1
1	1	1	0	0	0	0

NAND Gate

Bubbled OR Gate



Duality Theorem: Starting with a Boolean relation, you can derive another Boolean relation by –

1. Changing each OR sign to an AND sign
2. Changing each AND sign to an OR sign
3. Complementing any 0 or 1 appearing in the expression.

Example: 1. We say that, $A+0 = A$; the dual is, $A.1 = A$

2. Consider, $A(B+C) = AB + AC$

By changing the OR and AND operation, we get the dual relation:

$$A + BC = (A+B)(A+C)$$

Laws of Boolean Algebra:

✓ The following laws are of immense use in the simplification of Boolean expressions.



ANALOG AND DIGITAL ELECTRONICS

- ✓ Note that, if A is a variable, then either $A = 0$ or $A = 1$. Also, when $A = 0$, $A \neq 1$; and when $A = 1$, $A \neq 0$.

De Morgan's First Theorem:-

The complement of sum is equal to the product of the complements.

$$(A + B)' = A' \cdot B' \quad \text{i.e., a bubbled AND gate \& a NOR gate are equivalent.}$$

De Morgan's Second Theorem:-

The complement of a product is equal to the sum of the compliments.

$$(A \cdot B)' = A' + B' \quad \text{i.e., a bubbled OR gate \& a NAND gate are equivalent.}$$

1) *Commutative Law:-*

$$A + B = B + A \quad \text{and} \quad A \cdot B = B \cdot A$$

2) *Associative Law:-*

$$A + (B + C) = (A + B) + C \quad \text{and} \quad A \cdot (BC) = (AB) \cdot C$$

3) *Distributive Law:-*

$$A(B + C) = AB + AC$$

4) *In relation to OR operation, the following laws hold good:-*

$$A + 0 = A$$

$$A + A = A$$

$$A + 1 = 1 \quad \text{and}$$

$$A + A' = 1$$

5) *In relation to AND operation, the following laws hold good:-*

$$A \cdot 1 = A$$

$$A \cdot A = A$$

$$A \cdot 0 = 0$$

$$A \cdot A' = 0$$

$$A'' = A$$

6) *Some more useful Boolean relations:-*

$$A + AB = A$$

$$A + A'B = A + B$$

$$A(A + B) = A$$

$$A(A' + B) = AB$$

$$A + (B \cdot C) = (A + B)(A + C)$$



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Simplification of Boolean Expressions:-

❖ The following **hints** are found to be of use, in reducing complex Boolean expressions –

1. If there are parentheses present in the given expression, they are removed first; since, multiplication should precede addition.

$$\text{E.g.:} - AB + C(A + B) = AB + AC + BC$$

2. If there are several identical terms, all except one can be removed.

$$\text{E.g.:} - A + B + C + A \cdot 1 = A + B + C + A = A + B + C$$

3. If a variable repeats in a term, only one variable may be retained.

$$\text{E.g.:} - A \cdot A = A$$

$$B \cdot B \cdot C = BC$$

4. If in any term, both a variable & its complement are present, that term may be removed; since, $AA' = 0$.

$$\text{E.g.:} - XX'Y = 0 \cdot Y = 0$$

5. Identify pairs of terms which contains same variables. If in a pair, a variable is absent in one term, it can be removed.

$$\begin{aligned}\text{E.g.:} - ABCD + ABC &= ABC(D + 1) \\ &= ABC \cdot 1 \quad \text{since, } 1 + D = 1 \\ &= ABC\end{aligned}$$

6. If, in a pair of terms, several variables are common, and another variable is present in one term & its complement is present in another term, this variable & its complement can be removed.

$$\begin{aligned}\text{E.g.:} - ABC + A'BC &= BC(A' + A) \\ &= BC \cdot 1 \quad \text{since, } A' + A = 1 \\ &= BC\end{aligned}$$

KARNAUGH MAPS

MINIMUM FORMS OF SWITCHING FUNCTIONS:

When a function is realized using AND and OR gates, *the cost of realizing the function is directly related to the number of gates and gate inputs used.* The Karnaugh map techniques developed, lead directly to *minimum cost* two-level circuits composed of AND and OR gates. An expression consisting of a *sum-of-product* terms corresponds directly to a two-level circuit composed of a group of AND gates feeding a single OR gate (see the following Figure). Similarly, a *product-of-sums* expression corresponds to a two-level circuit composed of OR gates feeding a single AND gate.

Therefore, to find minimum cost two-level AND-OR gate circuits, we must find minimum expressions in sum-of-products or product-of-sums form.



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A *minimum sum-of-products* expression for a function is defined as a sum of product terms which

- has a minimum number of terms and
- of all those expressions which have the same minimum number of terms, has a minimum number of literals.

The minimum sum of products corresponds directly to a minimum two-level gate circuit which has

- a minimum number of gates and
- a minimum number of gate inputs.

Unlike the minterm expansion for a function, the minimum sum of products is not necessarily unique; that is, a given function may have two different minimum sum-of-products forms, each with the same number of terms and the same number of literals.

Given a minterm expansion, the minimum sum-of products form can often be obtained by the following procedure:

- Combine terms by using $XY' + XY = X(Y' + Y) = X$. Do this repeatedly to eliminate as many literals as possible. A given term may be used more than once because $X+X=X$.
- Eliminate redundant terms by using the theorems of Boolean Algebra.

Example

Find a minimum sum-of-products expression for

$$\begin{aligned} F(a, b, c) &= \sum m(0, 1, 2, 5, 6, 7) \\ F &= a'b'c' + a'b'c + a'bc' + ab'c + abc' + abc \\ &= a'b' + b'c + bc' + ab \end{aligned}$$

None of the terms in the above expression can be eliminated by consensus. However, combining terms in a different way leads directly to a minimum sum of products:

$$\begin{aligned} F &= a'b'c' + a'b'c + a'bc' + ab'c + abc' + abc \\ &= a'b' + bc' + ac \end{aligned}$$

A *minimum product-of-sums* expression for a function is defined as a product of sum terms which

- has a minimum number of factors, and
- of all those expressions which have the same number of factors, has a minimum number of literals.

Unlike the maxterm expansion, the minimum product-of-sums form of a function is not necessarily unique. Given a maxterm expansion, the minimum product of sums can often be obtained by a procedure similar to that used in the minimum sum-of-products case, except that the theorem $(XX + Y')(XX + Y) = XX$ is used to combine terms.



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Example

$$\begin{aligned}
 & (A + B' + C + D')(A + B' + C' + D')(A + B' + C' + D)(A' + B' + C' + D)(A + B + C' + D)(A' + B + C' + D) \\
 &= (A + B' + D') \quad (A + B' + C') \quad (B' + C' + D) \quad (B + C' + D) \\
 &= (A + B' + D') \quad (A + B' + C') \quad (C' + D) \\
 &= (A + B' + D')(C' + D) \quad \leftarrow \text{eliminate by consensus}
 \end{aligned}$$

A	B	C	Y – Fundamental Product	Min-term
0	0	0	0	m0
0	0	1	0	m1
0	1	0	0	m2
0	1	1	$1 - ABC$	m3
1	0	0	0	m4
1	0	1	$1 - \bar{A}\bar{B}$	m5
1	1	0	$1 - ABC$	m6
1	1	1	$1 - ABC$	m7

A	B	C	Y – Fundamental Sum	Max-term
0	0	0	$0 - A + B + C$	M0
0	0	1	$0 - A + B + \bar{C}$	M1
0	1	0	$0 - A + \bar{B} + C$	M2
0	1	1	1	M3
1	0	0	$0 - \bar{A} + B + C$	M4
1	0	1	1	M5
1	1	0	1	M6
1	1	1	1	M7

$Y_{SOP} = \bar{A}BC + \bar{A}\bar{B}C + ABC + ABC$
 $= \sum m(1, 2, 4, 7).$

$Y_{SOP} = AB + BC + AC$

SOP Circuit Diagram:

No. of Gates = 4
No. of Gate Inputs = 9

 $Y_{POS} = (A + B + C)(A + B + \bar{C})(A + \bar{B} + C)(\bar{A} + B + C)$
 $= \prod M(0, 1, 2, 4).$

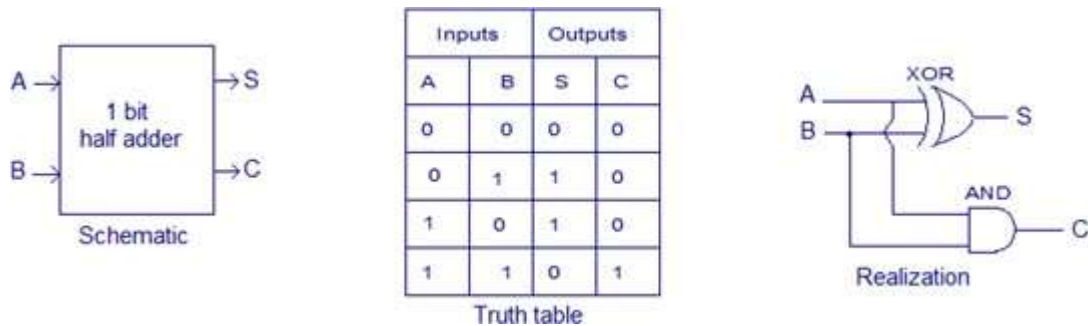
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Example: Adders & Subtractors

Adder circuit is a combinational digital circuit that is used for adding two numbers. A typical adder circuit produces a sum bit (denoted by S) and a carry bit (denoted by C) as the output. Adder circuits are of two types: Half adder and Full adder.

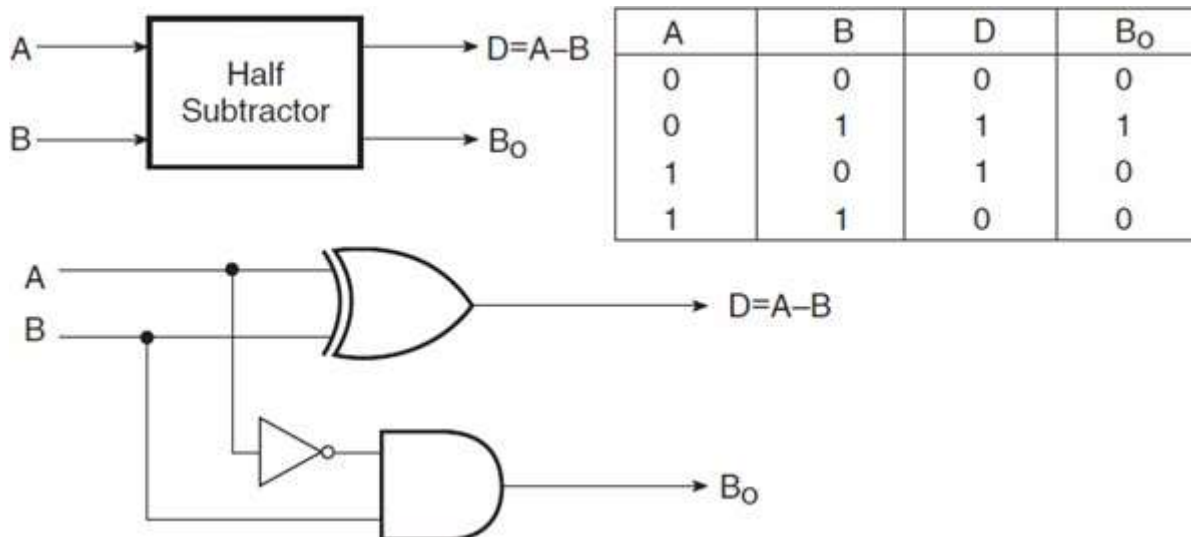
Subtractor is the one which is used to subtract two binary numbers (digits) and provides Difference and Borrow as an output.

Half Adder & Half Subtractor:



$$\text{Sum, } S = \bar{A}B + A\bar{B}$$

$$\text{Carry, } C = AB$$



$$\text{Difference, } D = \bar{A}B + A\bar{B}$$

$$\text{Borrow, } B_0 = \bar{A}B$$

Full Adder & Full Subtractor:

A	B	C _i	S	C _o	A	B	C _i	D	B _o
0	0	0	0	0	0	0	0	0	0
0	0	1	1	0	0	0	1	1	1
0	1	0	1	0	0	1	0	1	1
0	1	1	0	1	0	1	1	0	1
1	0	0	1	0	1	0	0	1	0

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1	0	1	0	1	1	0	1	0	0
1	1	0	0	1	1	1	0	0	0
1	1	1	1	1	1	1	1	1	1

Sum, $S = \sum m(1, 2, 4, 7) = \prod M(0, 3, 5, 6)$.

$$\text{Sum}, S = \bar{A}\bar{B}C + \bar{A}BC + A\bar{B}C + ABC$$

$$\text{Or}, S = \bar{C}(\bar{A}B + A\bar{B}) + C(\bar{A}B + AB)$$

$$\text{Or}, S = C(A \oplus B) + C(A \odot B)$$

$$\text{Or}, S = \bar{C}(A \oplus B) + C(\bar{A} \oplus \bar{B})$$

$$\text{Therefore}, S = A \oplus B \oplus C$$

Carry Out, $Co = \sum m(3, 5, 6, 7) = \prod M(0, 1, 2, 4)$.

$$\text{Carry Out}, Co = \bar{A}BC + \bar{A}\bar{B}C + ABC + ABC$$

$$\text{Or}, Co = \bar{C}(AB) + C(\bar{A}B + \bar{A}\bar{B} + AB)$$

$$\text{Therefore}, Co = AB + BC + AC$$

Sum

$\begin{matrix} C_{in} \\ AB \end{matrix}$	\bar{C}_{in}	C_{in}
$\bar{A}\bar{B}$		1
$\bar{A}B$	1	
AB		1
$A\bar{B}$	1	

Count

$\begin{matrix} C_{in} \\ AB \end{matrix}$	\bar{C}_{in}	C_{in}
$\bar{A}\bar{B}$		
$\bar{A}B$		1
AB	1	1
$A\bar{B}$		1

Difference, $D = \sum m(1, 2, 4, 7) = \prod M(0, 3, 5, 6)$.

$$\text{Difference}, D = \bar{A}\bar{B}C + \bar{A}BC + A\bar{B}C + ABC$$

$$\text{Or}, D = \bar{C}(\bar{A}B + A\bar{B}) + C(\bar{A}B + AB)$$

$$\text{Or}, D = C(A \oplus B) + C(A \odot B)$$

$$\text{Or}, D = \bar{C}(A \oplus B) + C(\bar{A} \oplus \bar{B})$$

$$\text{Therefore}, D = A \oplus B \oplus C$$

Borrow, $Bo = \sum m(1, 2, 3, 7) = \prod M(0, 4, 5, 6)$.

$$\text{Borrow}, Bo = \bar{A}\bar{B}C + \bar{A}BC + \bar{A}\bar{B}C + ABC$$

$$\text{Or}, Bo = \bar{C}(\bar{A}B) + C(\bar{A}\bar{B} + \bar{A}B + AB)$$

$$\text{Therefore}, Bo = \bar{A}B + BC + \bar{A}C$$

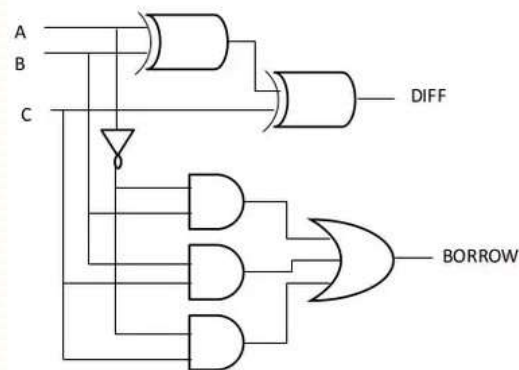
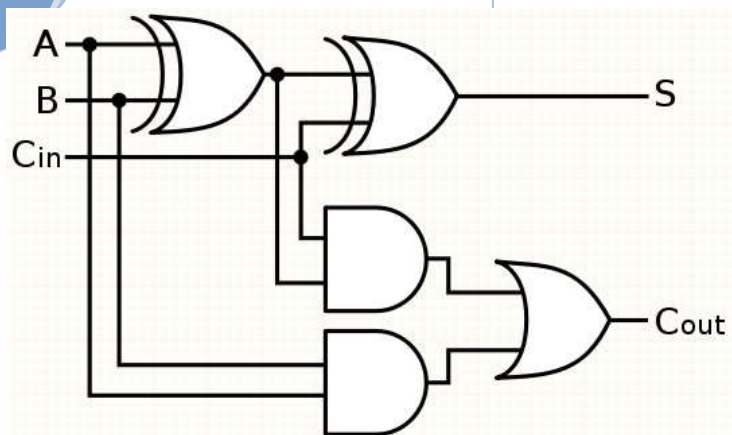
Difference

$\begin{matrix} C_{in} \\ AB \end{matrix}$	\bar{C}_{in}	C_{in}
$\bar{A}\bar{B}$		1
$\bar{A}B$	1	
AB		1
$A\bar{B}$	1	

Borrow

$\begin{matrix} C_{in} \\ AB \end{matrix}$	\bar{C}_{in}	C_{in}
$\bar{A}\bar{B}$		1
$\bar{A}B$	1	1
AB		1
$A\bar{B}$		





TWO AND THREE VARIABLE KARNAUGH MAPS:

Just like a truth table, the Karnaugh map of a function specifies the value of the function for every combination of values of the independent variables. The following Figure shows the truth table for a function F and the corresponding Karnaugh map:

A	B	F
0	0	1
0	1	1
1	0	0
1	1	0

(a)

A	B	
0	0	1
0	1	1
1	0	0
1	1	0

(b)

A	B	
0	0	1
0	1	1
1	0	0
1	1	0

$$F = A'B' + A'B$$

(c)

A	B	
0	0	1
0	1	1
1	0	0
1	1	0

$$F = A'$$

(d)

The following Figure shows a three-variable truth table and the corresponding Karnaugh map:

		a	
		0	1
bc	00	000	100
	01	001	101
	11	011	111
	10	010	110

100 is adjacent to 110

(a) Binary notation

a bc	0	1
00	0	4
01	1	5
11	3	7
10	2	6

(b) Decimal notation

c \ ab	00	01	11	10
0	000	010	110	100
1	001	011	111	101

(c) Binary notation

		c	
		0	1
ab	00	0	1
	01	2	3
	11	6	7
	10	4	5

(d) Decimal notation

Example: Write the Karnaugh Map for – (a) $f = \sum m(1, 3, 5)$ (b) $oo(a, b, c) = abc' + b'c + a'$
 (c) $F = \sum m(0, 1, 2, 5, 6, 7)$

Solution: (a) $f = \sum m(1, 3, 5)$ (b) $oo(a, b, c) = abc' + b'c + a'$ (c) $F = \sum m(0, 1, 2, 5, 6, 7)$

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	a	0	1
bc			
00		0	0
01		1	0
11		1	1
10		0	0

(a)

1. The term abc' is 1 when $a = 1$ and $bc = 10$, so we place a 1 in the square which corresponds to the $a = 1$ column and the $bc = 10$ row of the map.
2. The term $b'c$ is 1 when $bc = 01$, so we place 1's in both squares of the $bc = 01$ row of the map.
3. The term a' is 1 when $a = 0$, so we place 1's in all the squares of the $a = 0$ column of the map. (Note: Since there already is a 1 in the $abc = 001$ square, we do not have to place a second 1 there because $x + x = x$.)

	a	0	1
bc			
00		1	
01		1	1
11		1	
10		1	1

(b)

	a	0	1
bc			
00		1	
01		1	1
11			1
10		1	1

$$F = a'b' + bc' + ac$$

(c)

	a	0	1
bc			
00		1	
01		1	1
11			1
10		1	1

$$F = a'c' + b'c + ab$$

Example: Find two different minimum sum-of-products expressions for the function $G = \sum m(0, 2, 3, 4, 5, 7)$.

Solution: Given, $G = \sum m(0, 2, 3, 4, 5, 7)$;

	a	0	1
bc			
00		1	1
01			1
11		1	1
10		1	

G

	a	0	1
bc			
00		1	1
01			1
11		1	1
10		1	

G

$G =$ _____

$G =$ _____

FOUR-VARIABLE KARNAUGH MAPS:

The following Figure shows the location of minterms on a four-variable map & plot of four-variable expression $oo(a, b, c, d) = acd + a'b + d'$ on a Karnaugh map:

f	ϕ	ϕ	ab	ab
cd	0	4	12	8
cd	1	5	13	9
cd	3	7	15	11
cd	2	6	14	10

f	00	01	11	10
00				
01				
11				
10				

$f(a, b, c, d) =$

Example: Write the Karnaugh map for (a) $oo = x'z' + wxy + x'y$

(b) $Y = \bar{A}\bar{B} + \bar{A}BC + \bar{A}\bar{C} + ABC$

Solution:

f	$\bar{w}\bar{x}$	$w\bar{x}$	wx	$\bar{w}x$
$\bar{y}\bar{z}$				
$\bar{y}z$				
$y\bar{z}$				
yz				

Y	0	1
00		
01		
11		
10		

$$f(w, x, y, z) =$$

$$Y(A, B, C) =$$

Example: Write the Karnaugh map for (a) $f_1 = \sum m(3, 4, 5, 6, 7, 9, 12, 13)$; (b) $f_2 = \sum m(2, 3, 5, 7, 8, 10, 11, 15)$.

Solution: (a) Given, $f_1 = \sum m(3, 4, 5, 6, 7, 9, 12, 13)$ & $f_2 = \sum m(2, 3, 5, 7, 8, 10, 11, 15)$;

$\begin{matrix} ab \\ cd \end{matrix}$	00	01	11	10
00		1	1	
01		1	1	1
11	1	1		
10		1		

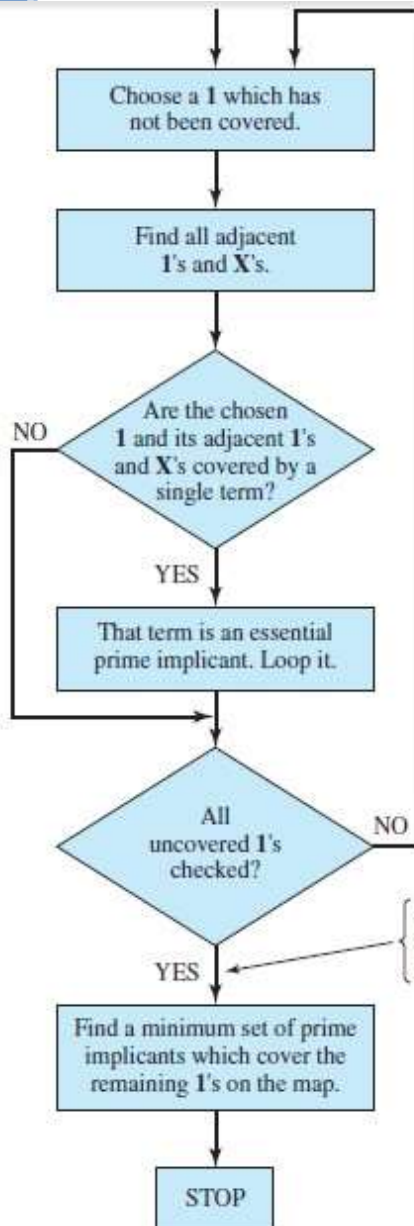
$\begin{matrix} ab \\ cd \end{matrix}$	00	01	11	10
00				1
01		1		
11	1	1	1	1
10	1			1

$$f_1 = \underline{\hspace{10em}} \quad f_2 = \underline{\hspace{10em}}$$

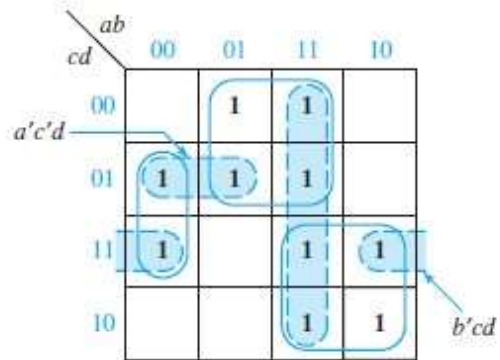
DETERMINATION OF MINIMUM EXPRESSIONS USING ESSENTIAL PRIME IMPLICANTS:

Any single 1 or any group of 1's which can be combined together on a map of the function F represents a product term which is called an implicant of F. Several implicants of F MAY BE POSSIBLE. A product term implicant is called a *prime implicant* if it cannot be combined with another term to eliminate a variable.

The following Figure shows the flowchart for determining a Minimum Sum of Products using a Karnaugh Map with an Example.



Minimum solution: $F = a'b'd + bc' + ac$
 All prime implicants: $a'b'd, bc', ac, a'c'd, ab, b'cd$



Note: All essential prime implicants have been determined at this point.

1. Choose a minterm (a 1) which has not yet been covered.
2. Find all 1's and X's adjacent to that minterm (Check the n adjacent squares on an n -variable map).
3. If a single term covers the minterm and all of the adjacent 1's and X's, then that term is an essential prime implicant, so select that term. (Note that don't-care terms are treated like 1's in steps 2 and 3 but not in step 1.)
4. Repeat steps 1, 2, and 3 until all essential prime implicants have been chosen.
5. Find a minimum set of prime implicants which cover the remaining 1's on the map. (If there is more than one such set, choose a set with a minimum number of literals.)

PAIRS, QUADS, AND OCTETS:

Pairs: The following K-map contains a pair of 1s that are horizontally adjacent. Two adjacent 1s, such as these are called a *pair*. A *pair eliminates one variable and its complement*.

Y	$\bar{A}B$	AB	AB	AB
$\bar{C}\bar{D}$	0	0	0	0
CD	0	0	0	0
CD	0	0	1	0
$C\bar{D}$	0	0	1	0

The sum-of-product equation is:

$$Y = ABCD + ABCD' = ABC(D + D') = ABC$$

Quad: A *quad* is a group of four 1s that are horizontally or vertically adjacent. A *quad eliminates two variables and their complements*.

Y	$\bar{A}B$	AB	AB	AB
$\bar{C}\bar{D}$	0	0	1	0
CD	0	0	1	0
CD	0	0	1	0
$C\bar{D}$	0	0	1	0

The sum-of-product equation is:

$$Y = ABC' + ABC = AB(C + C') = AB$$

The Octet: The *octet* is a group of eight 1s, as shown in the following Fig. An *octet eliminates three variables and their complements*.

Y	$\bar{A}B$	AB	AB	AB
$\bar{C}\bar{D}$	0	0	1	1
CD	0	0	1	1
CD	0	0	1	1
$C\bar{D}$	0	0	1	1

The sum-of-product equation is:

$$Y = AB + AB' = A(B + B') = A$$

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KARNAUGH SIMPLIFICATIONS:

A pair eliminates one variable and its complement. A quad eliminates two variables and their complements. An octet eliminates three variables and their complements. Because of this, after drawing the K-map, first encircle the octets, then the quads, and finally the pairs, to get highest simplification.

Example: Using K-map, simplify; $Y = \sum m(1, 2, 3, 6, 8, 9, 10, 12, 13, 14)$.

Solution:

Y	$\bar{A}\bar{B}$	$A\bar{B}$	AB	AB
$\bar{C}\bar{D}$	0	0	1	1
$C\bar{D}$	1	0	1	1
CD	1	0	0	0
$C\bar{D}$	1	1	1	1

$$Y = AC' + CD' + A'B'D$$

Overlapping Groups: Always overlap groups.

Y1	$\bar{A}\bar{B}$	$A\bar{B}$	AB	AB
$\bar{C}\bar{D}$	0	0	0	0
$C\bar{D}$	0	1	0	0
CD	1	1	1	1
$C\bar{D}$	1	1	1	1

Y1 =

Y2	$\bar{A}\bar{B}$	$A\bar{B}$	AB	AB
$\bar{C}\bar{D}$	0	0	0	0
$C\bar{D}$	0	1	0	0
CD	1	1	1	1
$C\bar{D}$	1	1	1	1

Y2 =

Rolling the Map:

Y3	$\bar{A}\bar{B}$	$A\bar{B}$	AB	AB
$\bar{C}\bar{D}$	0	0	0	0
$C\bar{D}$	1	0	0	1
CD	1	0	0	1
$C\bar{D}$	0	0	0	0

Y3 =

Y4	$\bar{A}\bar{B}$	$A\bar{B}$	AB	AB
$\bar{C}\bar{D}$	0	0	0	0
$C\bar{D}$	1	0	0	1
CD	1	0	0	1
$C\bar{D}$	0	0	0	0

Y4 =

Rolling and Overlapping:

Y1	$\bar{A}\bar{B}$	$\bar{A}B$	AB	$A\bar{B}$
$\bar{C}\bar{D}$	1	1	0	0
$\bar{C}D$	1	1	0	1
CD	1	1	0	1
$C\bar{D}$	1	1	0	0

Y1 =

Y2	$\bar{A}\bar{B}$	$\bar{A}B$	AB	$A\bar{B}$
$\bar{C}\bar{D}$	1	1	0	0
$\bar{C}D$	1	1	0	1
CD	1	1	0	1
$C\bar{D}$	1	1	0	0

Y2 =

	$\bar{A}\bar{B}$	$\bar{A}B$	AB	$A\bar{B}$
$\bar{C}\bar{D}$	1	1	0	1
$\bar{C}D$	1	1	0	1
CD	1	1	0	0
$C\bar{D}$	1	1	0	1

Y3 =

Y4	$\bar{A}\bar{B}$	$\bar{A}B$	AB	$A\bar{B}$
$\bar{C}\bar{D}$	1	1	0	1
$\bar{C}D$	1	1	0	1
CD	1	1	0	0
$C\bar{D}$	1	1	0	1

Y4 =

Y5	$\bar{A}\bar{B}$	$\bar{A}B$	AB	$A\bar{B}$
$\bar{C}\bar{D}$	1	1	0	1
$\bar{C}D$	1	1	0	1
CD	1	1	0	0
$C\bar{D}$	1	1	0	1

Y5 =

Eliminating Redundant Groups: After encircling groups, eliminate any *redundant groups*. This is a group whose 1s are already used by other groups.

	$\bar{A}\bar{B}$	$\bar{A}B$	AB	$A\bar{B}$
$\bar{C}\bar{D}$	0	0	1	0
$\bar{C}D$	1	1	1	0
CD	0	1	1	1
$C\bar{D}$	0	1	0	0

Y	$\bar{A}\bar{B}$	$\bar{A}B$	AB	$A\bar{B}$
$\bar{C}\bar{D}$	0	0	1	0
$\bar{C}D$	1	1	1	0
CD	0	1	1	1
$C\bar{D}$	0	1	0	0

Y	$\bar{A}\bar{B}$	$\bar{A}B$	AB	$A\bar{B}$
$\bar{C}\bar{D}$	0	0	1	0
$\bar{C}D$	1	1	1	0
CD	0	1	1	1
$C\bar{D}$	0	1	0	0

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- ✓ Two minterms will combine if they differ in exactly one variable. The examples given below show both the binary notation and its algebraic equivalent.

$$\begin{array}{c}
 AB'CD' + AB'CD = AB'C \\
 \underbrace{1\ 0\ 1\ 0} + \underbrace{1\ 0\ 1\ 1} = \underbrace{1\ 0\ 1} - \text{(the dash indicates a missing variable)} \\
 \quad X\ Y \quad \quad X\ Y' \quad X \\
 \\
 A'BC'D + A'BCD' \text{ (will not combine)} \\
 0\ 1\ 0\ 1 + 0\ 1\ 1\ 0 \text{ (will not combine)}
 \end{array}$$

- ✓ In order to find all of the prime implicants, all possible pairs of minterms should be compared and combined whenever possible. To reduce the required number of comparisons, the binary minterms are sorted into groups according to the number of 1's in each term.

Now, function; $f(a, b, c, d) = \sum m(0, 1, 2, 5, 6, 7, 8, 9, 10, 14)$ can be represented by following list of minterms:

- ✓ In this list, the term in group 0 has zero 1's, the terms in group 1 have one 1, those in group 2 have two 1's, and those in group 3 have three 1's.
- ✓ Two terms can be combined if they differ in exactly one variable. Only terms in adjacent groups must be compared.
- ✓ First, we will compare the term in group 0 with all of the terms in group 1. Terms 0000 and 0001 can be combined to eliminate the fourth variable, which yields $000 - (a'b'c')$.
- ✓ Similarly, 0 and 2 combine to form $00-0 (a'b'd')$, and 0 and 8 combine to form $-000 (b'c'd')$. The resulting terms are listed in Column II of the following Table.
- ✓ Whenever two terms combine, the corresponding decimal numbers differ by a power of 2 (1, 2, 4, 8, etc.).
- ✓ Since the comparison of group 0 with groups 2 and 3 is unnecessary, we proceed to compare terms in groups 1 and 2. Comparing term 1 with all terms in group 2, we find that it combines with 5 and 9 but not with 6 or 10. Similarly, term 2 combines only with 6 and 10, and term 8 only with 9 and 10. The resulting terms are listed in Column 2.
- ✓ Each time a term is combined with another term, it is checked off. Also note that, a term may be used more than once. Even though two terms have already been combined with other terms, they still must be compared and combined if possible.
- ✓ At this stage, we may generate redundant terms, but these redundant terms will be eliminated later.

group 0	0	0000
group 1	{	1 0001
		2 0010
		8 1000
group 2	{	5 0101
		6 0110
		9 1001
		10 1010
group 3	{	7 0111
		14 1110

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- ✓ We finish with Column 1 by comparing terms in groups 2 and 3. New terms are formed by combining terms 5 and 7, 6 and 7, 6 and 14, and 10 and 14.

a	b	c	d	f	<u>Column 1</u>	<u>Column 2</u>	<u>Column 3</u>
0	0	0	0	1	abcd	abcd	abcd
0	0	0	1	1			
0	0	1	0	1			
0	0	1	1	0			
0	1	0	0	0			
0	1	0	1	1			
0	1	1	0	1			
0	1	1	1	1			
1	0	0	0	1			
1	0	0	1	1			
1	0	1	0	1			
1	0	1	1	0			
1	1	0	0	0			
1	1	0	1	0			
1	1	1	0	1			
1	1	1	1	0			

- ✓ Note that the terms in Column 2 have been divided into groups. In order to combine two terms, the terms must have the same variables, and the terms must differ in exactly one of these variables. Thus, it is necessary only to compare terms which have dashes (missing variables) in corresponding places and which differ by exactly one in the number of 1's.
- ✓ Terms in the first group in Column 2 need only be compared with terms in the second group which have dashes in the same places. Term 000– (0, 1) combines only with term 100– (8, 9) to yield – 00– ($b'c'$).
- ✓ The resulting term is listed in Column 3 along with the designation 0, 1, 8, 9 to indicate that it was formed by combining minterms 0, 1, 8, and 9.
- ✓ Term (0, 2) combines only with (8, 10), and term (0, 8) combines with both (1, 9) and (2, 10).
- ✓ Again, the terms which have been combined are checked off. Comparing terms from the second and third groups in Column 2, we find that (2,6) combines with (10, 14), and (2, 10) combines with (6,14).
- ✓ Note that there are three pairs of duplicate terms in Column 3. These duplicate terms were formed in each case by combining the same set of four minterms in a different order.

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- ✓ After deleting the duplicate terms, we compare terms from the two groups in Column 3. Because no further combination is possible, the process terminates.
- ✓ In general, we would keep comparing terms and forming new groups of terms and new columns until no more terms could be combined. The terms which have not been checked off because they cannot be combined with other terms are called *prime implicants*. Because every minterm has been included in at least one of the prime implicants, the function is equal to the sum of its prime implicants. In this example we have;

$$f = a'c'd + a'bd + a'bc + b'c' + b'd' + cd'$$

(1, 5) (5, 7) (6, 7) (0, 1, 8, 9) (0, 2, 8, 10) (2, 6, 10, 14)

Definition:

- ✓ Given a function F of n variables, a product term P is an **implicant** of F iff for every combination of values of the n variables for which P = 1, F is also equal to 1.
- ✓ A **prime implicant** of a function F is a product term implicant which is no longer an implicant if any literal is deleted from it.

Consider an Example:

$$F(a, b, c) = a'b'c' + ab'c' + ab'c + abc = b'c' + ac$$

- In the above function, the implicant $a'b'c'$ is not a prime implicant because a can be eliminated, and the resulting term $b'c'$ is still an implicant of F. The implicants $b'c'$ and ac are prime implicants because if we delete a literal from either term, the term will no longer be an implicant of F.

The Quine-McCluskey method, as previously illustrated, finds all of the product term implicants of a function. The implicants which are nonprime are checked off in the process of combining terms, so that the remaining terms are prime implicants. Any nonprime term in a sum-of-products expression can thus be replaced with a prime implicant, which reduces the number of literals and simplifies the expression.

THE PRIME IMPLICANT CHART:

- ✓ The second part of the Quine-McCluskey method employs a prime implicant chart to select a minimum set of prime implicants. The minterms of the function are listed across the top of the chart, and the prime implicants are listed down the side. A prime implicant is equal to a sum of minterms, and the prime implicant is said to cover these minterms. If a prime implicant covers a given minterm, an **X** is placed at the intersection of the corresponding row and column. The



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following Table shows the prime implicant. All of the prime implicants (terms which have not been checked off in the above Table) are listed on the left.

-	0	1	2	5	6	7	8	9	10	14
(0, 1, 8, 9) ($b'c'$)										
(0, 2, 8, 10) ($b'd'$)										
(2, 6, 10, 14) (cd')										
(1, 5) ($a'c'd$)										
(5, 7) ($a'bd$)										
(6, 7) ($a'bc$)										

- ✓ In the first row, X's are placed in columns 0, 1, 8, and 9, because prime implicant $b'c'$ was formed from the sum of minterms 0, 1, 8, and 9. Similarly, the all other X's are placed.
- ✓ If a minterm is covered by only one prime implicant, then that prime implicant is called an *essential prime implicant* and must be included in the minimum sum of products. Essential prime implicants are easy to find using the prime implicant chart. If a given column contains only one X, then the corresponding row is an essential prime implicant. In the above Table, columns 9 and 14 each contain one X, so prime implicants $b'c'$ and cd' are essential.
- ✓ Each time a prime implicant is selected for inclusion in the minimum sum, the corresponding row should be crossed out. After doing this, the columns which correspond to all minterms covered by that prime implicant should also be crossed out.
- ✓ A minimum set of prime implicants must now be chosen to cover the remaining columns. In this example, the resulting minimum sum of products is –

$$f = b'c' + cd' + a'bd$$

Example: Solve using QM method: $F = \sum m(0, 1, 2, 5, 6, 7)$.

Solution:

a	b	c	F	<u>Column 1</u>	<u>Column 2</u>
1	0	0		abc	abc
1	0	1			
1	1	0			
1	1	1			
1	0	0			
1	0	1			
1	1	0			
1	1	1			



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The following Table shows the resulting prime implicants chart:

-	0	1	2	5	6	7

Therefore, $F =$

NOTE: A prime implicant chart which has two or more X 's in every column is called a *cyclic prime implicant chart*.

Example: Solve, using Quine Mc-Cluskey method & K-Map method: $F = \sum m (0, 1, 2, 8, 10, 11, 14, 15)$.

Solution:

Quine Mc-Clusky method:

A	B	C	D	F	Stage 1 ABCD	Stage 2 ABCD	Stage 3 ABCD
0	0	0	0				
0	0	0	1				
0	0	1	0				
0	0	1	1				
0	1	0	0				
0	1	0	1				
0	1	1	0				
0	1	1	1				
1	0	0	0				
1	0	0	1				
1	0	1	0				
1	0	1	1				
1	1	0	0				
1	1	0	1				
1	1	1	0				
1	1	1	1				



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-	0	1	2	8	10	11	14	15
(0, 1) $(A'B'C')$								
(0, 2, 8, 10) $(B'D')$								
(10, 11, 14, 15) (AC)								

Therefore, $F = AC + B'D' + A'B'C'$

K-Map Method:

F	$\bar{A}B$	AB	AB	AB
$\bar{C}\bar{D}$				
CD				
CD				
$C\bar{D}$				

$$F = AC + B'D' + A'B'C'$$

Homework: Using Quine-McClusky method, simplify;

a) $Y = \sum m(0, 1, 2, 3, 10, 11, 12, 13, 14, 15)$

b) $Y = \sum m(2, 6, 7).$

PATRICK'S METHOD:

- ✓ Patrick's method is a technique for determining all minimum sum-of-products solutions from a prime implicant chart. The example discussed above has two minimum solutions. As the number of variables increases, the number of prime implicants and the complexity of the prime implicant chart may increase significantly. In such cases, a large amount of trial and error may be required to find the minimum solution(s).
- ✓ Patrick's method is a more systematic way of finding all minimum solutions from a prime implicant chart than the method used previously. Before applying Patrick's method, all essential prime implicants and the minterms they cover should be removed from the chart.

Consider the following Table:



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- ✓ First, we will label the rows of the table P1, P2, P3, etc. We will form a logic function, P, which is true when all of the minterms in the chart have been covered. Let P1 be a logic variable which is true when the prime implicant in row P1 is included in the solution, P2 be a logic variable which is true when the prime implicant in row P2 is included in the solution, etc.
 - ✓ Since, column 0 has X's in rows P1 and P2, we must choose row P1 or P2 in order to cover minterm 0. Therefore, the expression (P1+P2) must be true.
 - ✓ In order to cover minterm 1, we must choose row P1 or P3; therefore, (P2+P3) must be true. In order to cover minterm 2, (P2+P4) must be true.
 - ✓ Similarly, in order to cover minterms 5, 6, and 7, the expressions (P3+P5), (P4+P6) and (P5+P6) must be true.
 - ✓ Since we must cover all of the minterms, the following function must be true:

 - ✓ The next step is to reduce P to a minimum sum-of-products. This is easy because there are no complements. First, we multiply out, using $(XX + Y)(XX + Z) = XX + YZ$ and the ordinary distributive law:

 - ✓ Next we use $XX + XXY = XX$ to eliminate redundant terms from P, which gives;

 - ✓ Because P must be true ($P = 1$) in order to cover all of the minterms, we can translate the equation back into words as follows. In order to cover all of the minterms, we must choose rows P1 and P4 and P5, or rows P1 and P2 and P5 and P6, or . . . or rows P2 and P3 and P6.
 - ✓ Although there are five possible solutions, only two of these have the minimum number of rows. Thus, the two solutions with the minimum number of prime implicants are obtained by choosing rows P1, P4, and P5 or rows P2, P3, and P6.
- or $F = a'c' + b'c + ab$ are two minimum solutions.
- ✓ Thus; $F = a'b' + bc' + ac$ or

SIMPLIFICATION OF INCOMPLETELY SPECIFIED FUNCTIONS:

In some digital systems, certain input conditions never occur during normal operation; therefore, the corresponding output never appears. Since the output never appears, it is indicated by an X in the truth table.

The X is called a *don't-care condition*.

Remember these points about don't-care conditions:

1. Given the truth table, draw the K-map and transfer 0s, 1s, and don't-care terms.
2. Encircle the actual 1s on the K-map in the largest groups you can find treating don't cares as 1s.
3. After the actual 1s have been included in the groups, disregard the remaining don't cares by visualizing them as 0s.

Example: Consider the following truth table with don't care conditions for all the inputs from 1010 to 1111.

A	B	C	D	Y	A	B	C	D	Y
0	0	0	0	0	1	0	0	0	0
0	0	0	1	0	1	0	0	1	1
0	0	1	0	0	1	0	1	0	X
0	0	1	1	0	1	0	1	1	X
0	1	0	0	0	1	1	0	0	X
0	1	0	1	0	1	1	0	1	X
0	1	1	0	0	1	1	1	0	X
0	1	1	1	0	1	1	1	1	X

Y	$\bar{A}\bar{B}$	$\bar{A}B$	AB	$A\bar{B}$
$\bar{C}\bar{D}$				
$\bar{C}D$				
$C\bar{D}$				
CD				

Y =

Problem: What is the simplest logic circuit for –

- a) $Y1 = F(A, B, C, D) = \sum m(0) + \sum d(8, 9, 10, 11, 14, 15)$
- b) $Y2 = F(A, B, C, D) = \sum m(0) + \sum d(12, 13, 14, 15)$
- c) $Y3 = F(A, B, C, D) = \sum m(7) + \sum d(10, 11, 12, 13, 14, 15)$.

Solution:

(a)

	$\bar{A}\bar{B}$	$\bar{A}B$	AB	$A\bar{B}$
$\bar{C}\bar{D}$				
$\bar{C}D$				
$C\bar{D}$				
CD				

Therefore, $Y1 = B'C'D'$



(b)

Therefore, $Y2 = A'B'C'D'$

Y2	$\bar{A}\bar{B}$	$\bar{A}B$	AB	$A\bar{B}$
$\bar{C}\bar{D}$				
$\bar{C}D$				
CD				
$C\bar{D}$				

(c)

Y3 $\bar{A}\bar{B}$ $\bar{A}B$ AB $A\bar{B}$

$\bar{C}\bar{D}$				
$\bar{C}D$				
CD				
$C\bar{D}$				

Therefore, $Y3 = BCD$

Don't-Care Conditions in Quine McCluskey Method:

- In the process of finding the prime implicants, we will treat the don't-care terms as if they were required minterms. In this way, they can be combined with other minterms to eliminate as many literals as possible. When forming the prime implicant chart, the don't-cares are not listed at the top.
- This way, when the prime implicant chart is solved, all of the required minterms will be covered by one of the selected prime implicants. However, the don't-care terms are not included in the final solution unless they have been used in the process of forming one of the selected prime implicants.

Homework: Using *Quine-McCluskey method* (same questions can be asked to solve by using *Patrick's method* also), simplify;

a) $f(a, b, c, d) = \sum m(3, 4, 5, 7, 10, 12, 14, 15) + \sum d(2)$

b) $f(a, b, c, d) = \sum m(1, 5, 7, 9, 11, 12, 14, 15)$

c) $f(a, b, c, d) = \sum m(0, 1, 3, 5, 6, 7, 8, 10, 14, 15)$

d) $f(a, b, c, d) = \sum m(1, 3, 4, 5, 6, 7, 10, 12, 13) + \sum d(2, 9, 5)$

e) $f(a, b, c, d) = \sum m(9, 12, 13, 15) + \sum d(1, 4, 5, 7, 8, 11, 14)$.

Example: Solve using *QM method*: $F(A, B, C, D) = \sum m(2, 3, 7, 9, 11, 13) + \sum d(1, 10, 15)$.

Solution: The don't-care terms are treated like required minterms when finding the prime implicants:



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A	B	C	D	F	<u>Column 1</u> ABCD	<u>Column 2</u> ABCD	<u>Column 3</u> ABCD
0	0	0	0				
0	0	0	1				
0	0	1	0				
0	0	1	1				
0	1	0	0				
0	1	0	1				
0	1	1	0				
0	1	1	1				
1	0	0	0				
1	0	0	1				
1	0	1	0				
1	0	1	1				
1	1	0	0				
1	1	0	1				
1	1	1	0				
1	1	1	1				

The don't-care columns are omitted when forming the prime implicant chart:

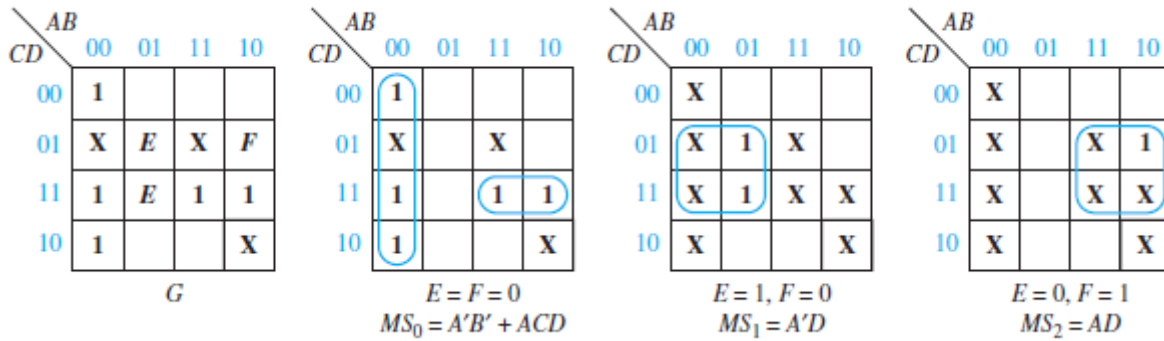
-	2	3	7	9	11	13

Therefore, $F =$

SIMPLIFICATION USING MAP-ENTERED VARIABLES:

Although the Quine-McCluskey method can be used with functions with a fairly large number of variables, it is not very efficient for functions that have many variables and relatively few terms. Some of these functions can be simplified by using a modification of the Karnaugh map method. By using map- entered variables, Karnaugh map techniques can be extended to simplify functions with more than four or five variables. The following Figure shows a four-variable map with two additional variables entered in the squares in the map.





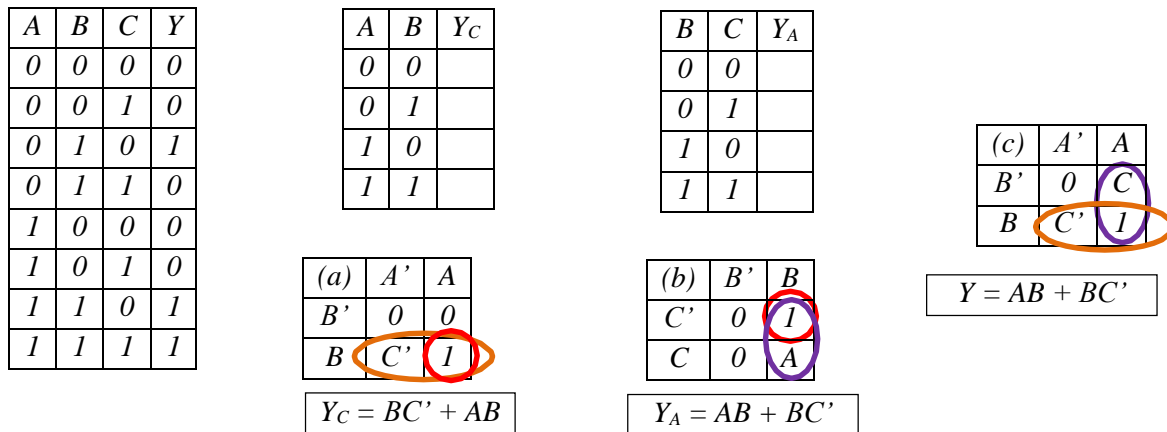
When E appears in a square, this means that if $E = 1$, the corresponding minterm is present in the function G , and if $E = 0$, the minterm is absent. Thus, the map represents the six-variable function;

$$G(A, B, C, D, E, F) = m_0 + m_2 + m_3 + Em_5 + Em_7 + Fm_9 + m_{11} + m_{15} \quad (+ \text{ don't-care terms})$$

Example: Simplify $Y(A, B, C) = \sum m(2, 6, 7)$ by using entered variable map method by taking –

- “ C ” as map entered variable
- “ A as map entered variables.

Solution: Let $Y = \sum m(2, 6, 7)$



Simplification is similar to K-map method. In Fig (a), C' is grouped with 1 to get a larger group as 1 can be written as $1 = 1 + C'$. Similarly, A is grouped with 1 in Fig (b).

Now, the product term representing each group is obtained by including map entered variable (MEV) in the group as an additional ANDed term.

Hence, for Fig (a): $Y = BC + AB$. For Fig (b): $Y = BC + AB$.

Consider the EBM shown in Fig (c). This has only two product terms; and doesn't need a separate coverage for 1. This is because, one can write $1 = C + C'$, and C is included in one group and C' is included in other group.

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Example: Simplify $Y(A, B, C) = \sum m(1, 2, 3, 4, 8, 9, 10, 13, 14)$ by using entered variable map method by taking –

- “D” as map entered variable
- “C and D” as map entered variables.

Solution:

A	B	C	D	Y
0	0	0	0	0
0	0	0	1	1
0	0	1	0	1
0	0	1	1	1
0	1	0	0	1
0	1	0	1	0
0	1	1	0	0
0	1	1	1	0
1	0	0	0	1
1	0	0	1	1
1	0	1	0	1
1	0	1	1	0
1	1	0	0	0
1	1	0	1	1
1	1	1	0	1
1	1	1	1	0

A	B	C	Y
0	0	0	D
0	0	1	1
0	1	0	\bar{D}
0	1	1	0
1	0	0	1
1	0	1	\bar{D}
1	1	0	D
1	1	1	\bar{D}

		BC			
A		00	01	11	10
	0	D	1	0	\bar{D}
	1	1	\bar{D}	\bar{D}	D

Compressed one time

A	B	Y
0	0	$C+D$
0	1	$\overline{C+D}$
1	0	$\overline{C \cdot D}$
1	1	$C \oplus D$

		B	
A		0	1
	0	$C+D$	$\overline{C+D}$
	1	$\overline{C \cdot D}$	$C \oplus D$

Compressed two times

Example: Solve by using (a) K-Map method & (b) MEV method taking “D” as map entered variable:

$F(A, B, C, D) = A' B' C + A' BC + A' BC' D + ABCD + (AB' C)$; where $AB' C$ is a don't-care term.

Solution: Given $F(A, B, C, D) = A' B' C + A' BC + A' BC' D + ABCD + (AB' C)$

i.e., $F = A' B' C(D + D') + A' BC(D + D') + A' BC' D + ABCD + [AB' C(D + D')]$

or $F = A' B' C' D' + A' B' C' D + A' BCD' + A' BCD + ABCD + (AB' CD' + AB' CD)$

i.e., $F = \sum m(0, 1, 6, 7, 15) + \sum d(10, 11)$.

Y	$\bar{A}\bar{B}$	$\bar{A}B$	$A\bar{B}$	AB
$\bar{C}\bar{D}$				
$\bar{C}D$				
CD				
$C\bar{D}$				

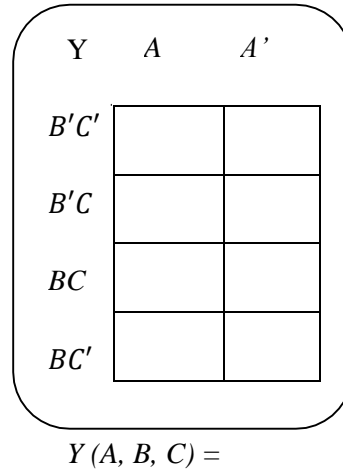
$Y(A, B, C, D) =$



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MEV method taking “D” as map entered variable:

A	B	C	D	Y	Y _D
0	0	0	0	1	1
0	0	0	1	1	
0	0	1	0	0	0
0	0	1	1	0	
0	1	0	0	0	0
0	1	0	1	0	
0	1	1	0	1	1
0	1	1	1	1	
1	0	0	0	0	0
1	0	0	1	0	
1	0	1	0	X	X
1	0	1	1	X	
1	1	0	0	0	0
1	1	0	1	0	
1	1	1	0	0	D
1	1	1	1	1	



Exercise:

- Design (a) Binary-to-Gray Code Converter, and (b) Gray-to-Binary Code Converter
- A switching circuit has two control inputs (C1 and C2), two data inputs (X1 and X2), and one output (Z). The circuit performs one of the logic operations AND, OR, EQU (equivalence), or XOR (exclusive OR) on the two data inputs. The function performed depends on the control inputs:

- Derive a truth table for Z

C ₁	C ₂	Function Performed by Circuit
0	0	OR
0	1	XOR
1	0	AND
1	1	EQU

- Use a Karnaugh Map to find minimum AN-OR Gate Circuit to realize Z.
- A logic circuit realizing the function f has four inputs a, b, c, d. The three inputs a, b, and c are the binary representation of the digits 0 through 7 with a being the most significant bit. The input

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d is an odd-parity bit; that is, the value of d is such that a, b, c, and d always contains an odd number of 1's. (For example, the digit 1 is represented by $abc = 001$ and $d = 0$, and the digit 3 is represented by $abcd = 0111$.) The function f has value 1 if the input digit is a prime number. (A number is prime if it is divisible only by itself and 1; 1 is considered to be prime, and 0 is not.)

- a. Draw a Karnaugh map for f*
- b. Find all prime implicants of f*
- c. Find all minimum sum of products for f*
- d. Find all prime implicants of f'*
- e. Find all minimum product of sums for f .*

Try these: Design a minimal sum combinational circuit to –

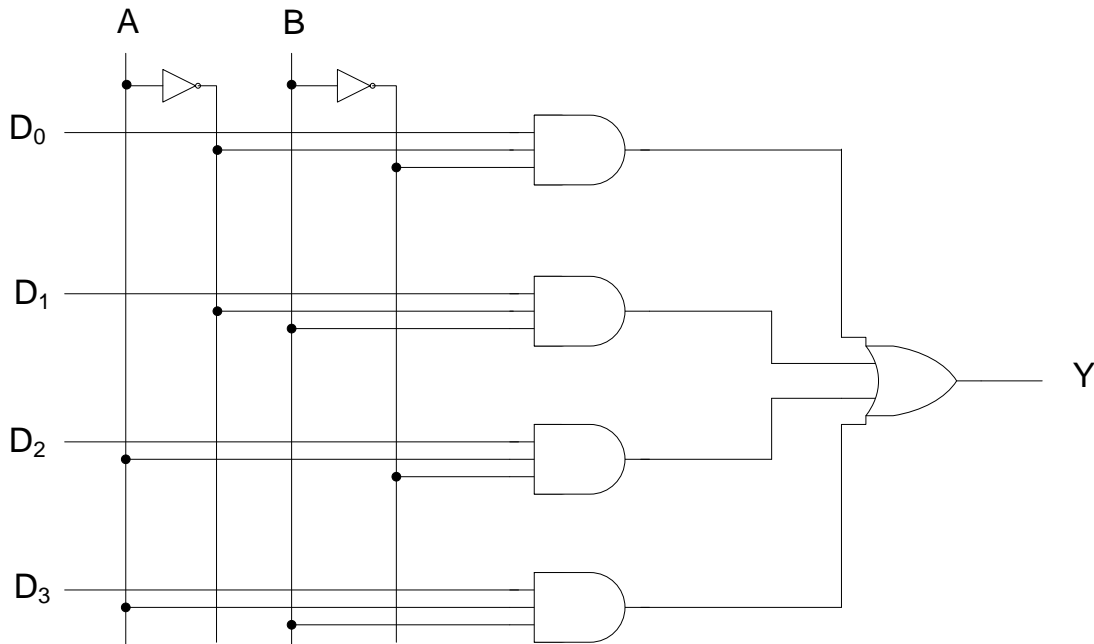
- a) Find the 9s complement of BCD numbers*
- b) Convert BCD to Excess-3*
- c) Multiply two 2-bit numbers*
- d) Output a 1 when an illegal BCD code occurs*
- e) Output the 2s complement of a 4-bit binary number.*



Q1. What is a multiplexer? Design a 4 to 1 multiplexer using logic gates. Write the truth table and explain its working principle.

Answer: Multiplexer is a circuit with many inputs but only one output.

Designing of 4 to 1 multiplexer shown below:



Truth table for 4 to 1 MUX:

A	B	Y
0	0	D_0
0	1	D_1
1	0	D_2
1	1	D_3

Working principle of 4 to 1 multiplexer:

From the above diagram, the Logic Equation for 4 to 1 multiplexer is

$$Y = A'B'D_0 + A'BD_1 + AB'D_2 + ABD_3$$

If $A=0$, $B=0$ then, $Y = 0'0'D_0 + 0'0D_1 + 00'D_2 + 00D_3 = 1.1.D_0 + 1.0.D_1 + 0.1.D_2 + 0.0.D_3 = D_0$

Similarly, if $A=0$ and $B=1$ then $Y=D_1$, if $A=1$ and $B=0$ then $Y=D_2$ and, if $A=1$ and $B=1$ then $Y=D_3$

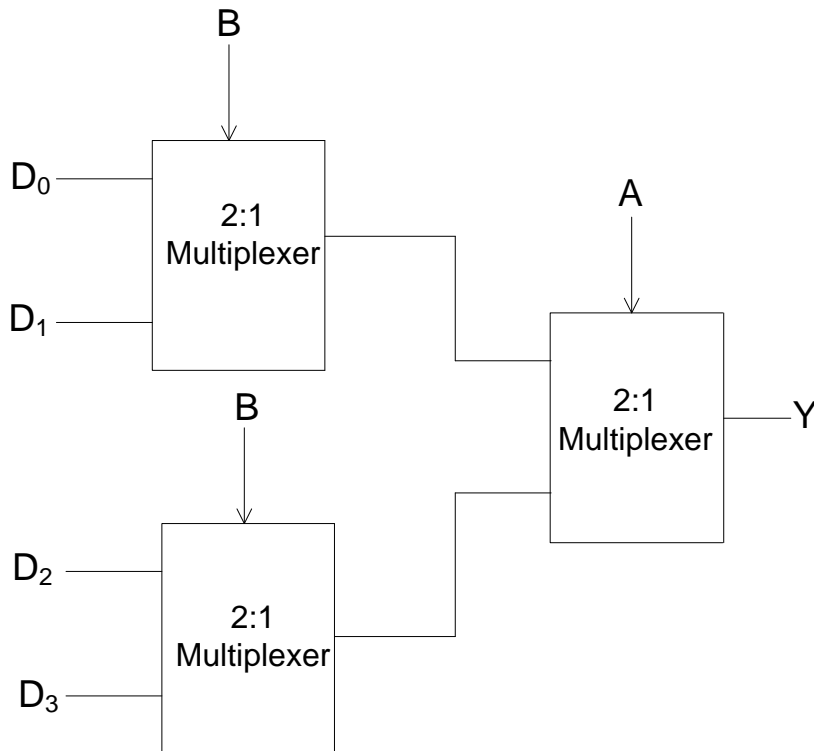
Q2. Construct 4:1 multiplexer using only 2:1 multiplexer.

Answer: *Logic Equation for 2:1 Multiplexer is $Y = A'D_0 + AD_1$*

Logic equation for 4:1 Multiplexer is $Y = A'B'D_0 + A'BD_1 + AB'D_2 + ABD_3$

$$\Rightarrow Y = A'(B'D_0 + BD_1) + A(B'D_2 + BD_3)$$

We require three 2:1 Multiplexers and the connection is shown below.



Q3. Construct 8:1 multiplexer using only 2:1 multiplexer.

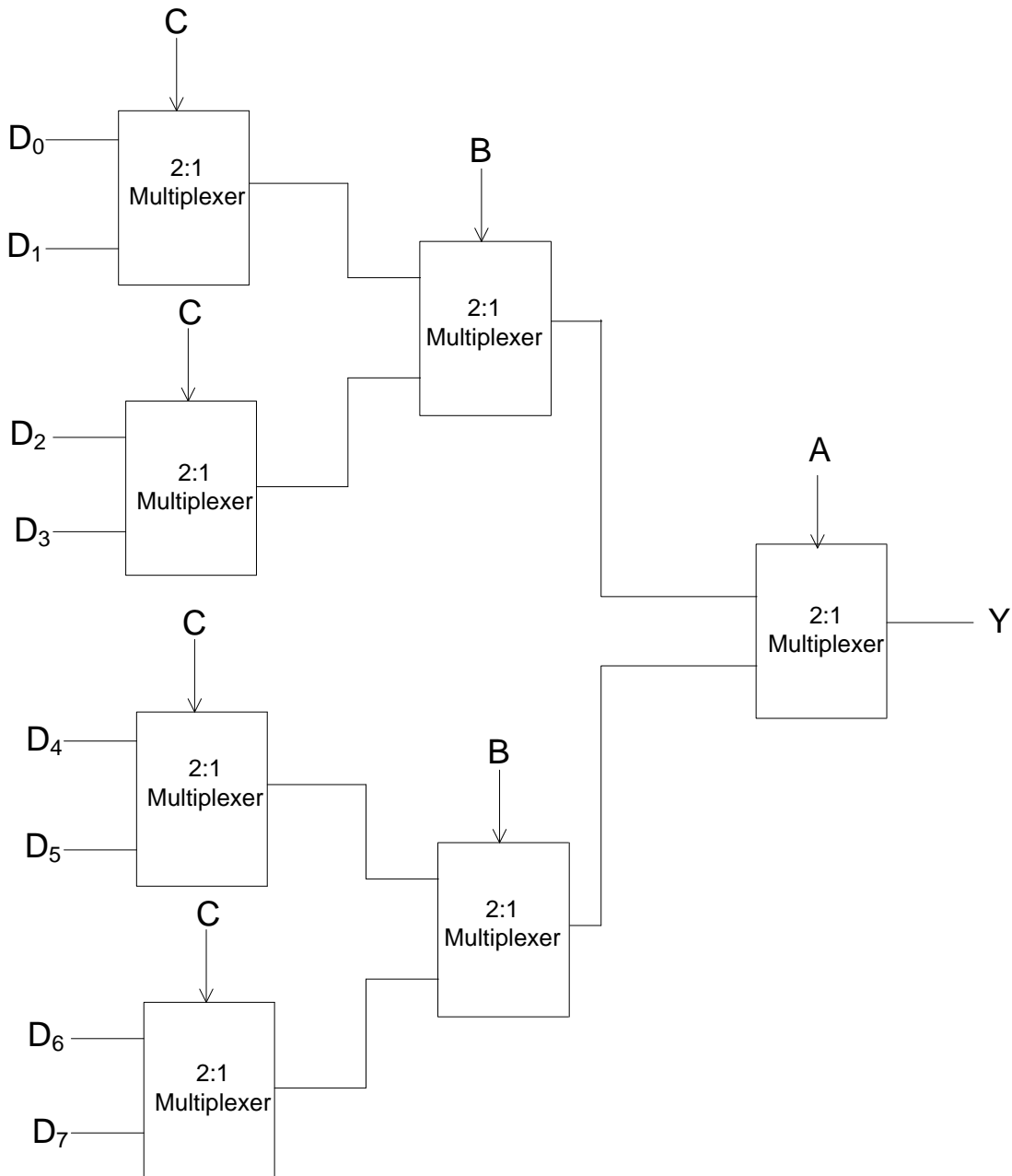
Answer: *Logic Equation for 2:1 Multiplexer is $Y = A'D_0 + AD_1$*

Logic Equation for 8:1 Multiplexer is

$$Y = A'B'C'D_0 + A'B'CD_1 + A'BC'D_2 + A'BCD_3 + AB'C'D_4 + AB'CD_5 + ABC'D_6 + ABCD_7$$

$$\Rightarrow Y = A'(B'C'D_0 + B'CD_1 + BC'D_2 + BCD_3) + A(B'C'D_4 + B'CD_5 + BC'D_6 + BCD_7)$$

$$\Rightarrow Y = A'[B'(C'D_0 + CD_1) + B(C'D_2 + CD_3)] + A[B'(C'D_4 + CD_5) + B(C'D_6 + CD_7)]$$



Q4. Design 16 to 1 multiplexer using 8 to 1 multiplexer and one 2 to 1 multiplexer.

Answer: *Logic Equation for 2:1 Multiplexer is* $Y = A'D_0 + AD_1$

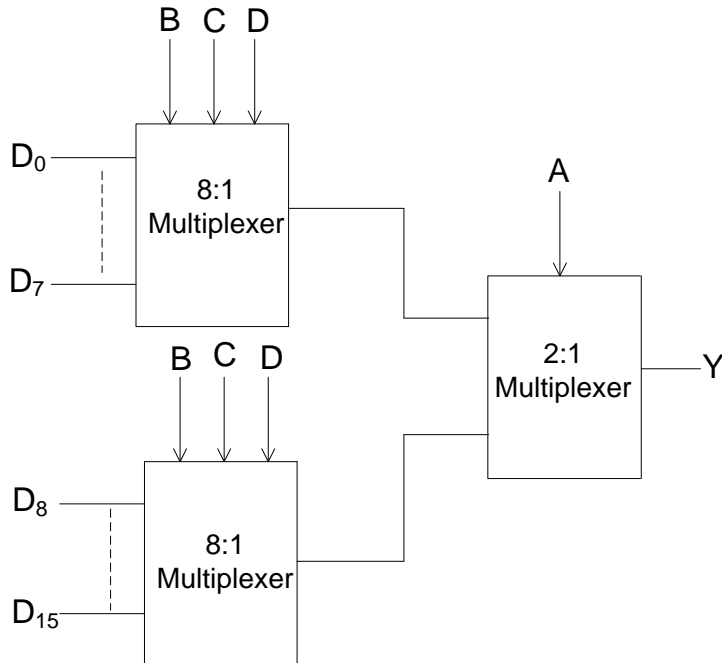
Logic Equation for 8:1 Multiplexer is

$$Y = A'B'C'D_0 + A'B'CD_1 + A'BC'D_2 + A'BCD_3 + AB'C'D_4 + AB'CD_5 + ABC'D_6 + ABCD_7$$

Logic Equation for 16:1 Multiplexer is

$$Y = A'B'C'D'D_0 + A'B'C'DD_1 + \dots + A'BCD'D_6 + A'BCDD_7 + AB'C'D'_8 + AB'C'D_9 + \dots + ABCD'D_{14} + ABCDD_{15}$$

$$\Rightarrow Y = A'(B'C'D'D_0 + B'C'DD_1 + \dots + BCD'D_6 + BCDD_7) + A(B'C'D'_8 + B'C'D_9 + \dots + BCD'D_{14} + BCDD_{15})$$



Q5. Design 32 to 1 multiplexer using 16 to 1 multiplexer and one 2 to 1 multiplexer.

Answer: *Logic Equation for 2:1 Multiplexer is $Y = A'D_0 + AD_1$*

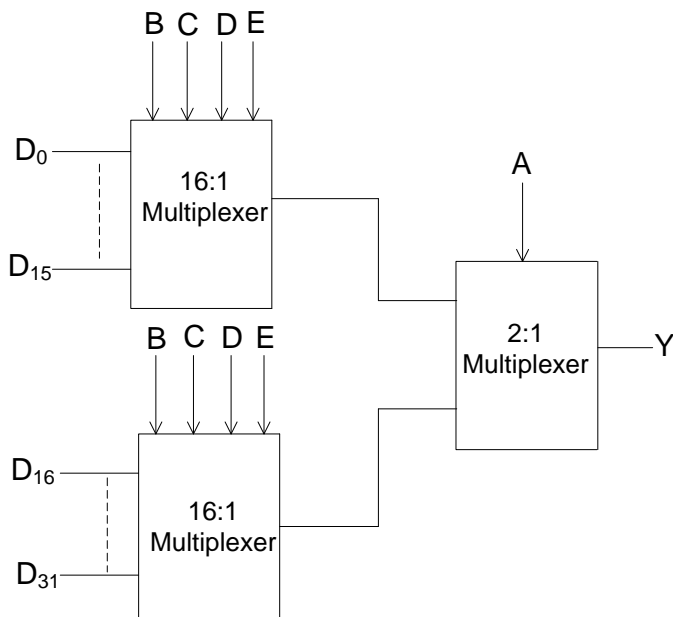
Logic Equation for 16:1 Multiplexer is

$$Y = A'B'C'D'D_0 + A'B'C'DD_1 + \dots + ABCD'D_{14} + ABCDD_{15}$$

Logic Equation for 32:1 Multiplexer is

$$Y = A'B'C'D'E'D_0 + \dots + A'BCDE'D_{14} + A'BCDED_{15} + AB'C'D'E'D_{16} + \dots + ABCDE'D_{30} + ABCDED_{31}$$

$$\Rightarrow Y = A'(B'C'D'E'D_0 + \dots + BCDE'D_{14} + BCDED_{15}) + A(B'C'D'E'D_{16} + \dots + BCDE'D_{30} + BCDED_{31})$$



Q6. Mention the differences between decoder and demultiplexer.

Answer:

Demultiplexer	Decoder
There is one data input and multiple output. There are selects used as control bits.	There is no data input. The only inputs are the control bit.
The data input appears at one of the output as per the control inputs.	One of the output is high as per the control inputs.
Input appears at the output where subscription of the output is equal to the decimal equivalent to the inputs.	Output becomes high where subscription of the output is equal to the decimal equivalent to the inputs

Q7. (a) Realize $Y = A'B + B'C' + ABC$ using an 8 to 1 Multiplexer.

(b) Can it be realized with a 4 to 1 multiplexer?

Answer: (a) Logic Equation for 8:1 Multiplexer is

$$Y = A'B'C'D_0 + A'B'CD_1 + A'BC'D_2 + A'BCD_3 + AB'C'D_4 + AB'CD_5 + ABC'D_6 + ABCD_7$$

We should express Y as a function of three variables i.e function of minterms.

$$Y = A'B + B'C' + ABC$$

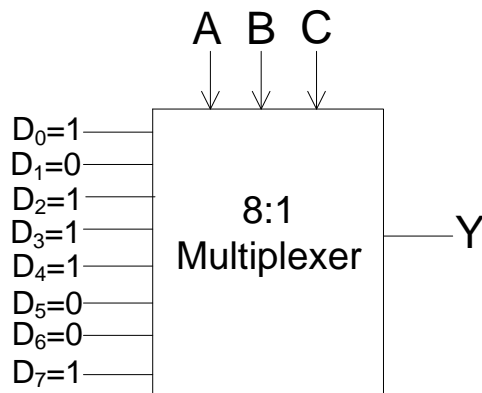
$$\Rightarrow Y = A'B(C + C') + B'C'(A + A') + ABC$$

$$\Rightarrow Y = A'BC + A'BC' + AB'C' + A'B'C' + ABC$$

$$\Rightarrow Y = A'B'C' + A'BC' + A'BC + AB'C' + ABC$$

Comparing with the Logic equation of 8:1 Multiplexer, we have

$$D_0 = D_2 = D_3 = D_4 = D_7 = 1 \text{ and } D_1 = D_5 = D_6 = 0$$



$$(b) Y = A'B + B'C' + ABC$$

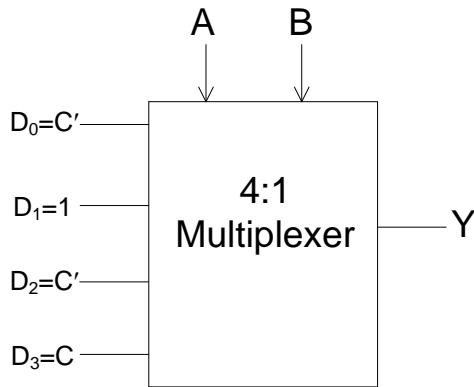
$$\Rightarrow Y = A'B + B'C'(A + A') + ABC$$

$$\Rightarrow Y = A'B + AB'C' + A'B'C' + ABC$$

$$\Rightarrow Y = A'B'.C' + A'B.1 + AB'.C' + AB.C$$

Logic equation for 4:1 Multiplexer is $Y = A'B'D_0 + A'BD_1 + AB'D_2 + ABD_3$

We have, $D_0 = C'$, $D_1 = 1$, $D_2 = C'$ and $D_3 = C$



Q8. Implement the following Boolean functions using 4:1 multiplexer (MUX):

(i) $Y = f(A, B, C, D) = \sum m(0, 1, 2, 4, 6, 9, 12, 14)$

(ii) $F(A, B, C) = \sum m(1, 3, 5, 6)$

Answer: Logic equation for 4:1 Multiplexer is $Y = A'B'D_0 + A'BD_1 + AB'D_2 + ABD_3$

$$Y = f(A, B, C, D) = \sum m(0, 1, 2, 4, 6, 9, 12, 14)$$

$$\Rightarrow Y = A'B'C'D' + A'B'C'D + A'B'CD' + A'BC'D' + A'BCD' + AB'C'D + ABC'D' + ABCD'$$

$$\Rightarrow Y = A'B'(C'D' + C'D + CD') + A'B(C'D' + CD') + AB'C'D + AB(C'D' + CD')$$

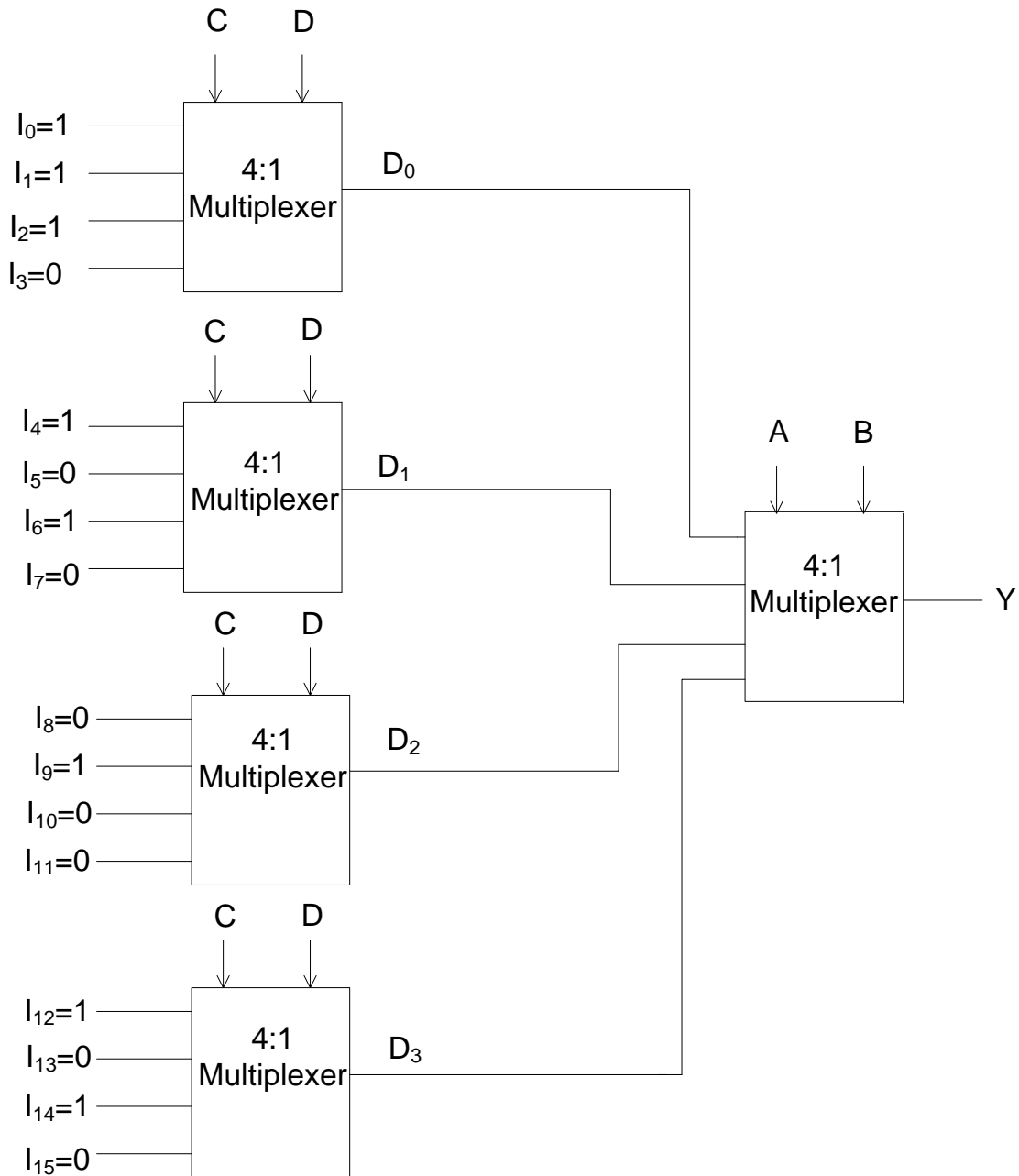
Comparing with Logic Equation of 4:1 Multiplexer, We have

$$D_0 = C'D' + C'D + CD' \Rightarrow D_0 = C'D'.1 + C'D.1 + CD'.1 + CD.0$$

$$D_1 = C'D' + CD' \Rightarrow D_1 = C'D'.1 + C'D.0 + CD'.1 + CD.0$$

$$D_2 = C'D \Rightarrow D_2 = C'D'.0 + C'D.1 + CD'.0 + CD.0$$

$$D_3 = C'D' + CD' \Rightarrow D_3 = C'D'.1 + C'D.0 + CD'.1 + CD.0$$



(ii) Logic equation for 4:1 Multiplexer is $Y = A'B'D_0 + A'BD_1 + AB'D_2 + ABD_3$

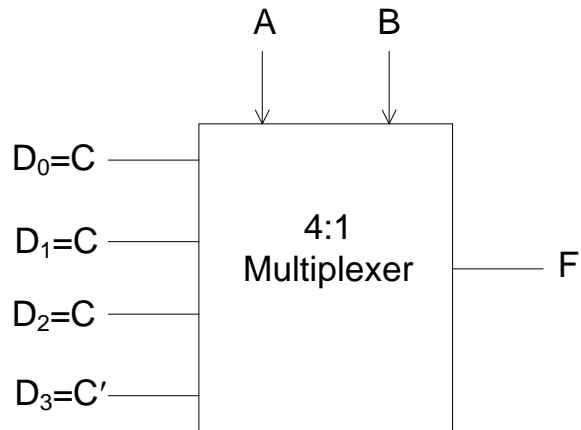
$$F(A, B, C) = \sum m(1, 3, 5, 6)$$

$$\Rightarrow F = A'B'C + A'BC + AB'C + ABC'$$

$$\Rightarrow F = A'B'.C + A'B.C + AB'.C + AB.C'$$

Comparing with Logic Equation of 4:1 Multiplexer, we have

$$D_0 = C, D_1 = C, D_2 = C \text{ and } D_3 = C'$$



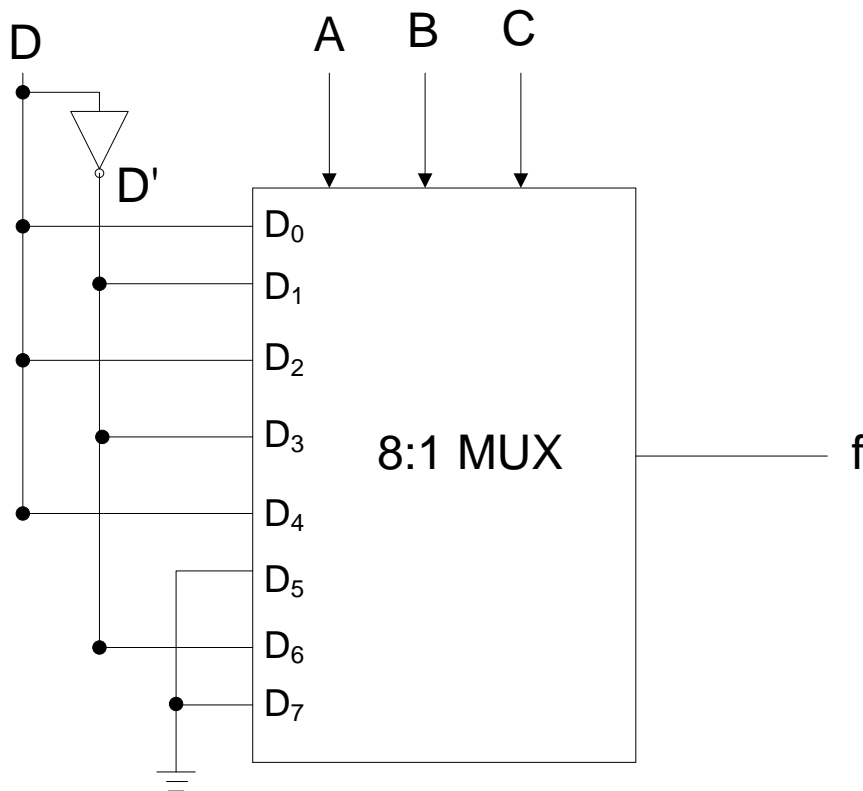
Q9. Implement the Boolean function expressed by SOP:

$$f(A, B, C, D) = \sum m(1, 2, 5, 6, 9, 12) \text{ using } 8 \text{ to } 1 \text{ MUX.}$$

Answer:

ABC	000	001	010	011	100	101	110	111
D=0	0	1	0	1	0	0	1	0
D=1	1	0	1	0	1	0	0	0
f	1	0	d	d'	d'	d'	d'	d
8:1 MUX data input	D ₀ =D	D ₁ = D'	D ₂ =D	D ₃ =D'	D ₄ = D	D ₅ = 0	D ₆ = D'	D ₇ =0

Circuit diagram:



Q10. Implement the Boolean function:

$F(A, B, C, D) = \sum m(0, 1, 2, 4, 5, 7, 8, 9)$ using 8 to 1 multiplexers. Draw the logic diagram and explain the operation. Additional gates can be used if required.

Answer : (a) Logic Equation for 8:1 Multiplexer is

$$Y = A'B'C'D_0 + A'B'CD_1 + A'BC'D_2 + A'BCD_3 + AB'C'D_4 + AB'CD_5 + ABC'D_6 + ABCD_7$$

Logic Equation for 2:1 Multiplexer is $Y = A'D_0 + AD_1$

$$F(A, B, C, D) = \sum m(0, 1, 2, 4, 5, 7, 8, 9)$$

$$\Rightarrow F = A'B'C'D' + A'B'C'D + A'B'CD' + A'BC'D' + A'BC'D + A'BCD + AB'C'D' + AB'C'D$$

$$\Rightarrow F = A'(B'C'D' + B'C'D + B'CD' + BC'D' + BC'D + BCD) + A(B'C'D' + B'C'D)$$

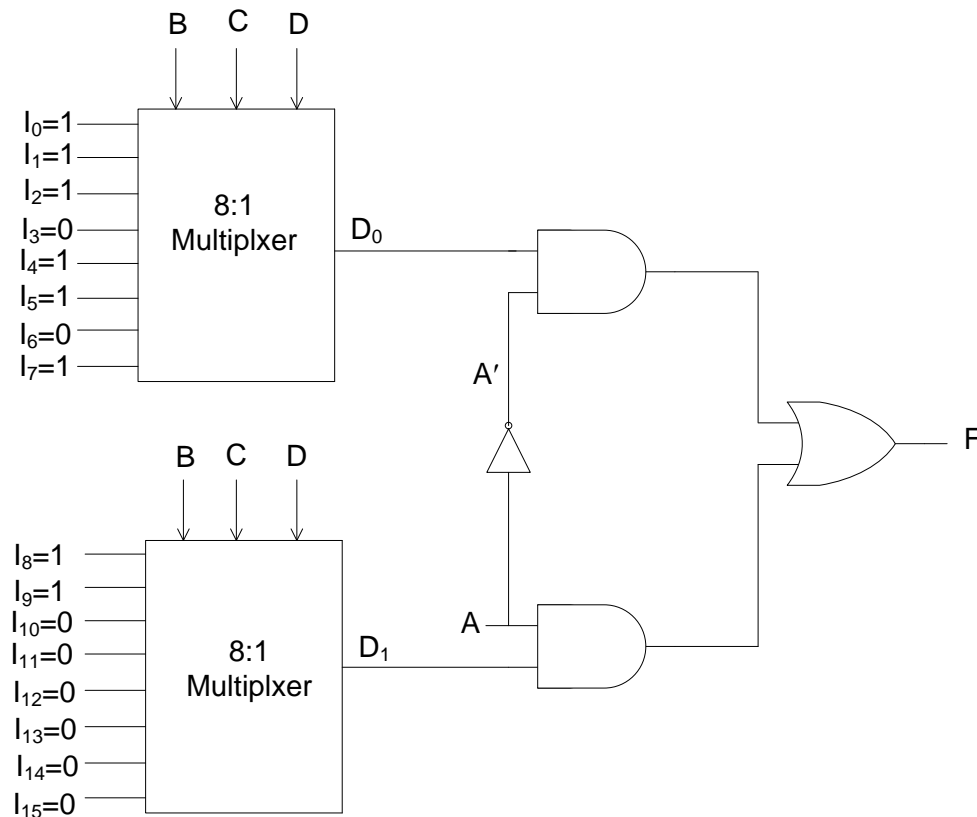
Comparing with 2:1 Logic Equation, we have

$$D_0 = B'C'D' + B'C'D + B'CD' + BC'D' + BC'D + BCD$$

$$\Rightarrow D_0 = B'C'D'.1 + B'C'D.1 + B'CD'.1 + B'CD.0 + BC'D'.1 + BC'D.0 + BCD'.0 + BCD.1$$

$$D_1 = B'C'D' + B'C'D$$

$$\Rightarrow D_1 = B'C'D'.1 + B'C'D.1 + B'CD'.0 + B'CD.0 + BC'D'.0 + BC'D.0 + BCD'.0 + BCD.0$$



(Note: Q9 and Q10 are similar. But method for Q9 is preferable)

Q11. Realize the following Boolean function:

$$P = f(w, x, y, z) = \sum(0, 1, 5, 6, 7, 10, 15) \text{ using (i) 16:1 MUX (ii) 8:1 MUX (iii) 4:1 MUX}$$

Answer : (i) Logic Equation for 16:1 Multiplexer is

$$P = w'x'y'z'.D_0 + w'x'y'z.D_1 + w'x'yz'.D_2 + w'x'yz.D_3 + w'xy'z'.D_4 + w'xy'z.D_5 + w'xyz'.D_6 + w'xyz.D_7 + wx'y'z'.D_8 + wx'y'z.D_9 + wx'yz'.D_{10} + wx'yz.D_{11} + wxy'z'.D_{12} + wxy'z.D_{13} + wxyz'.D_{14} + wxyz.D_{15}$$

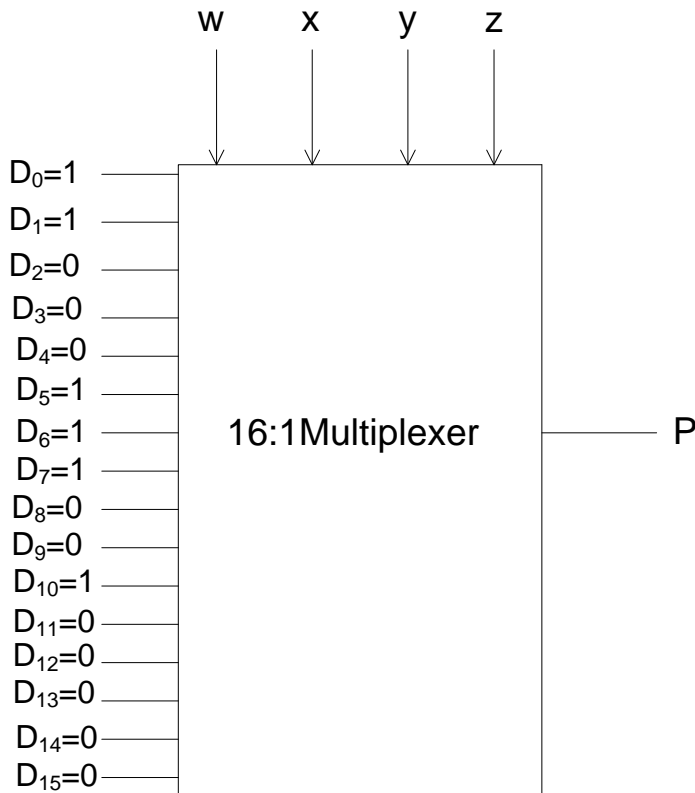
$$P = f(w, x, y, z) = \sum m(0, 1, 5, 6, 7, 10, 15)$$

$$\Rightarrow P = w'x'y'z' + w'x'y'z + w'xy'z' + w'xyz' + w'xyz + wx'y'z' + wxyz$$

$$\Rightarrow P = w'x'y'z'.1 + w'x'y'z.1 + w'x'yz'.0 + w'x'yz.0 + w'xy'z'.0 + w'xy'z.1 + w'xyz'.1 + w'xyz.1 + wx'y'z'.0 + wx'y'z.0 + wx'yz'.1 + wx'yz.0 + wxy'z'.0 + wxy'z.0 + wxyz'.0 + wxyz.1$$

Comparing with Logic Equation for 16:1 Multiplexer, we have

$$D_0 = D_1 = D_5 = D_6 = D_7 = D_{10} = D_{15} = 1 \text{ and } D_2 = D_3 = D_4 = D_8 = D_9 = D_{11} = D_{12} = D_{13} = D_{14} = 0$$



(ii) Logic Equation for 8:1 Multiplexer is

$$Y = x'y'z'.I_0 + x'y'z.I_1 + x'yz'.I_2 + x'yz.I_3 + xy'z'.I_4 + xy'z.I_5 + xyz'.I_6 + xyz.I_7$$

Logic Equation for 2:1 Multiplexer is $Y = w'D_0 + wD_1$

$$P = f(w, x, y, z) = \sum m(0, 1, 5, 6, 7, 10, 15)$$

$$\Rightarrow P = w'x'y'z' + w'x'y'z + w'xy'z' + w'xyz' + w'xyz + wx'y'z' + wxyz$$

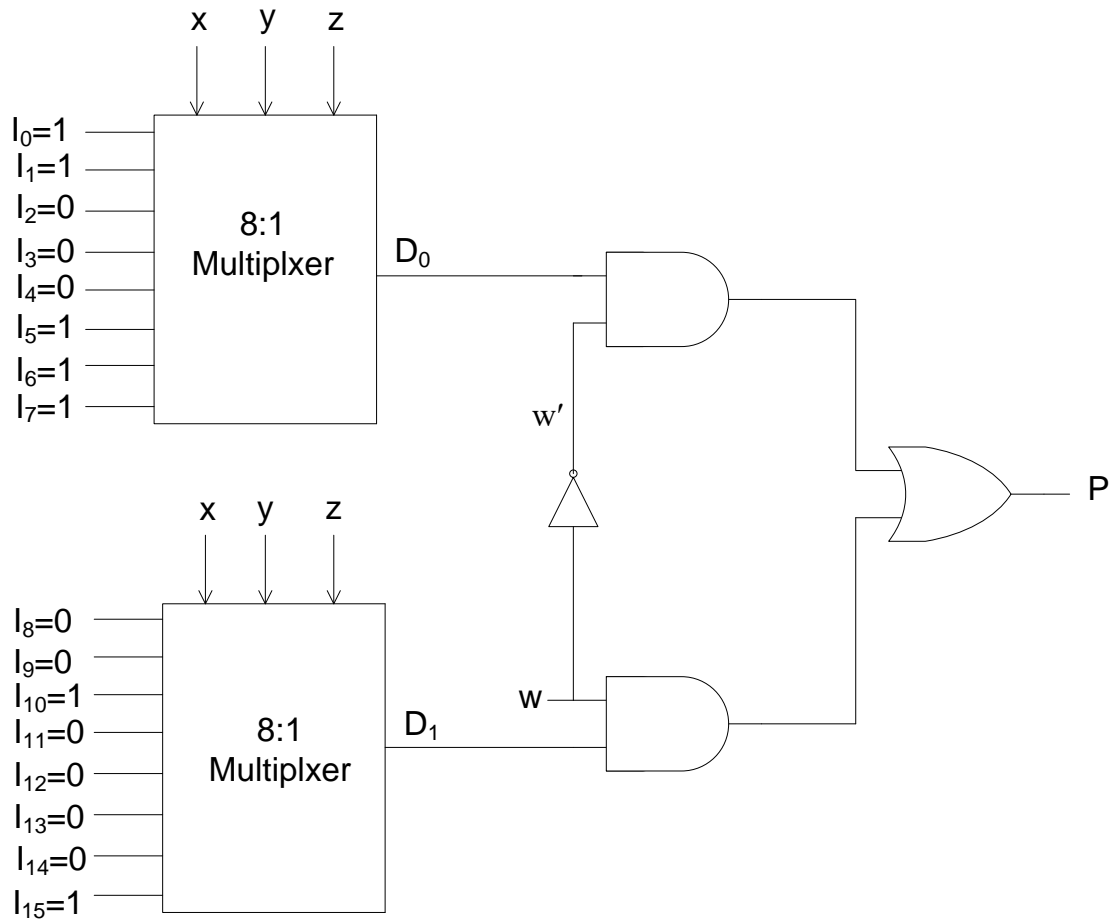
$$\Rightarrow P = w'(x'y'z' + x'y'z + xy'z' + xyz' + xyz) + w(x'y'z' + xyz)$$

$$\Rightarrow P = w'(x'y'z'.1 + x'y'z.1 + x'yz'.0 + x'yz.0 + xy'z'.0 + xy'z.1 + xyz'.1 + xyz.1) \\ + w(x'y'z'.0 + x'y'z.0 + x'yz'.1 + x'yz.0 + xy'z'.0 + xy'z.0 + xyz'.0 + xyz.1)$$

Comparing with Logic Equation of 2:1 Multiplexer, we have

$$D_0 = x'y'z'.1 + x'y'z.1 + x'yz'.0 + x'yz.0 + xy'z'.0 + xy'z.1 + xyz'.1 + xyz.1$$

$$D_1 = x'y'z'.0 + x'y'z.0 + x'yz'.1 + x'yz.0 + xy'z'.0 + xy'z.0 + xyz'.0 + xyz.1$$



(iii) Logic equation for 4:1 Multiplexer is $Y = w'x'D_0 + w'xD_1 + wx'D_2 + wxD_3$

$$P = f(w, x, y, z) = \sum m(0,1,5,6,7,10,15)$$

$$\Rightarrow P = w'x'y'z' + w'x'y'z + w'xy'z + w'xyz' + w'xyz + wx'y'z' + wxyz$$

$$\Rightarrow P = w'x'(y'z' + y'z) + w'x(y'z + yz' + yz) + wx'y'z' + wxyz$$

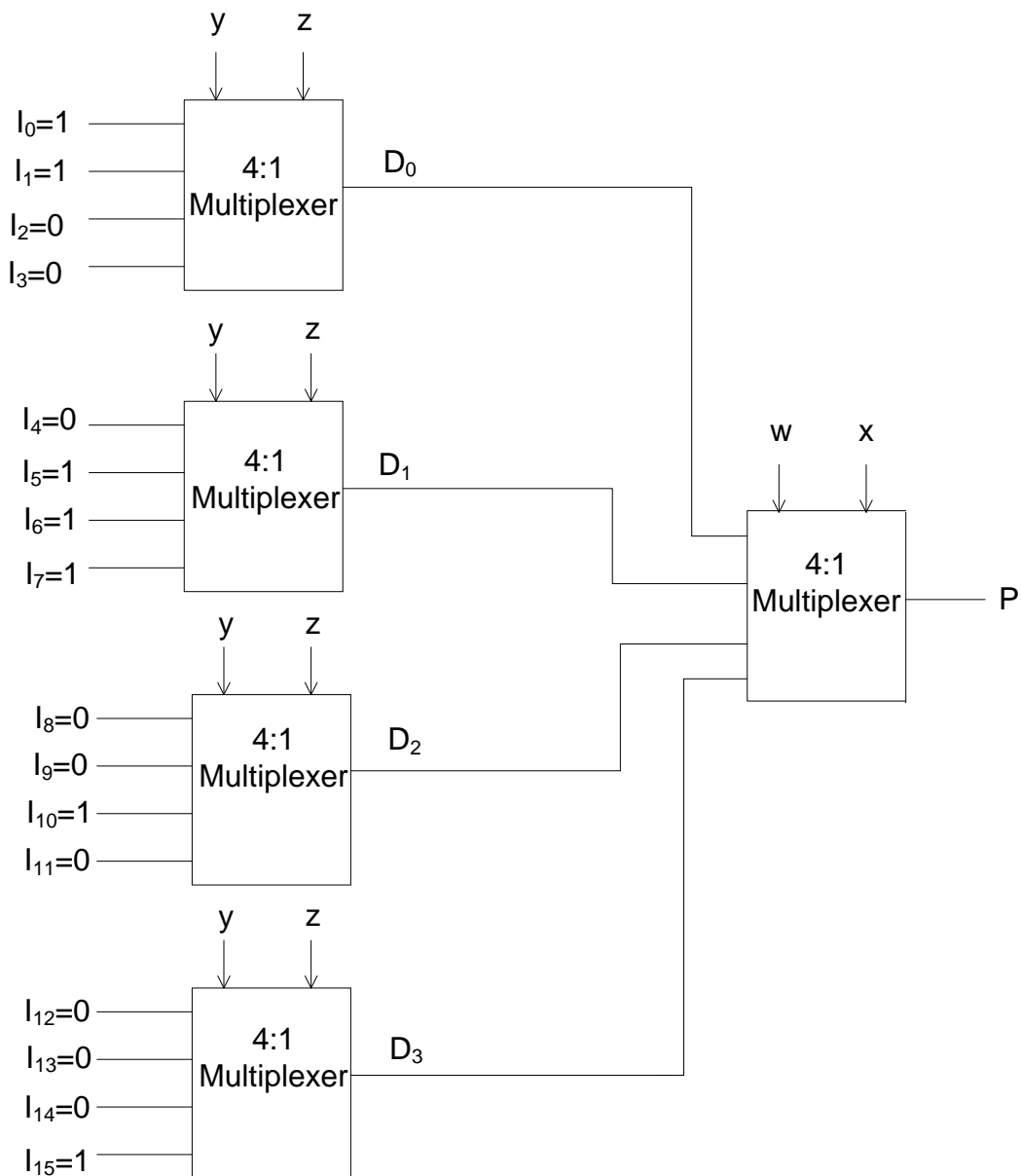
Comparing with Logic Equation for 4:1 Multiplexer, we have

$$D_0 = y'z' + y'z \Rightarrow D_0 = y'z'.1 + y'z.1 + yz'.0 + yz.0$$

$$D_1 = y'z + yz' + yz \Rightarrow D_1 = y'z'.0 + y'z.1 + yz'.1 + yz.1$$

$$D_2 = yz' = y'z'.0 + y'z.0 + yz'.1 + yz.0$$

$$D_3 = yz = y'z'.0 + y'z.0 + yz'.0 + yz.1$$



Q12. Design and implement BCD to excess-3 code converter using four 8:1 multiplexers. Take MSB 'A' as map entered variable(input variable) 'BCD' lines as select lines, assuming $f(A,B,C,D)$ as BCD input.

Answer: Truth table for converting BCD to Excess-3

BCD				Excess-3			
A	B	C	D	W	X	Y	Z
0	0	0	0	0	0	1	1
0	0	0	1	0	1	0	0
0	0	1	0	0	1	0	1
0	0	1	1	0	1	1	0
0	1	0	0	0	1	1	1
0	1	0	1	1	0	0	0
0	1	1	0	1	0	0	1
0	1	1	1	1	0	1	0
1	0	0	0	1	0	1	1
1	0	0	1	1	1	0	0

Designing of the multiplexer whose output is W

BCD	000	001	010	011	100	101	110	111
A=0	0	0	0	0	0	1	1	1
A=1	1	1	X	X	X	X	X	X
W	A	A	0	0	0	1	1	1
8:1 MUX Data Input	D ₀ =A	D ₁ =A	D ₂ =0	D ₃ =0	D ₄ =0	D ₅ =1	D ₆ =1	D ₇ =1

Designing of the multiplexer whose output is X

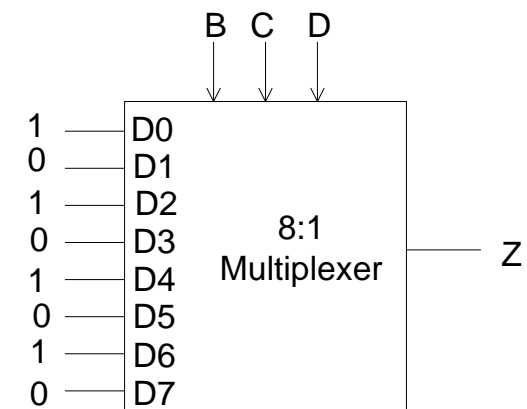
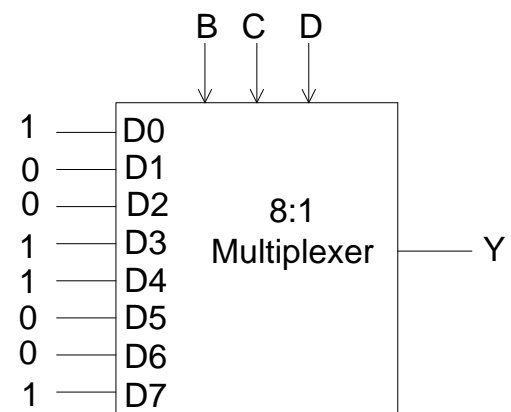
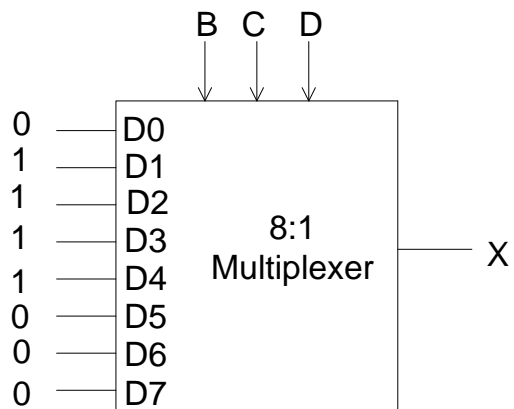
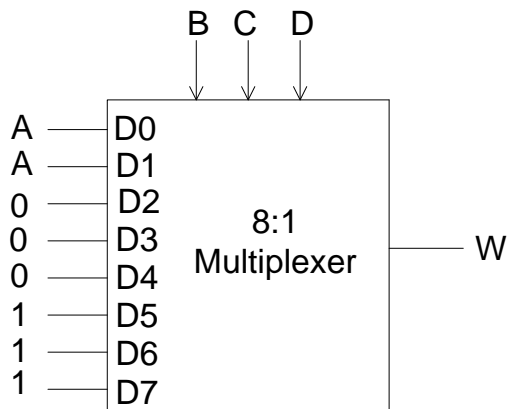
BCD	000	001	010	011	100	101	110	111
A=0	0	1	1	1	1	0	0	0
A=1	0	1	X	X	X	X	X	X
X	0	1	1	1	1	0	0	0
8:1 MUX Data Input	D ₀ =0	D ₁ =1	D ₂ =1	D ₃ =1	D ₄ =1	D ₅ =0	D ₆ =0	D ₇ =0

Designing of the multiplexer whose output is Y

BCD	000	001	010	011	100	101	110	111
A=0	1	0	0	1	1	0	0	1
A=1	1	0	X	X	X	X	X	X
Y	1	0	0	1	1	0	0	1
8:1 MUX Data Input	D ₀ =1	D ₁ =0	D ₂ =0	D ₃ =1	D ₄ =1	D ₅ =0	D ₆ =0	D ₇ =1

Designing of the multiplexer whose output is Z

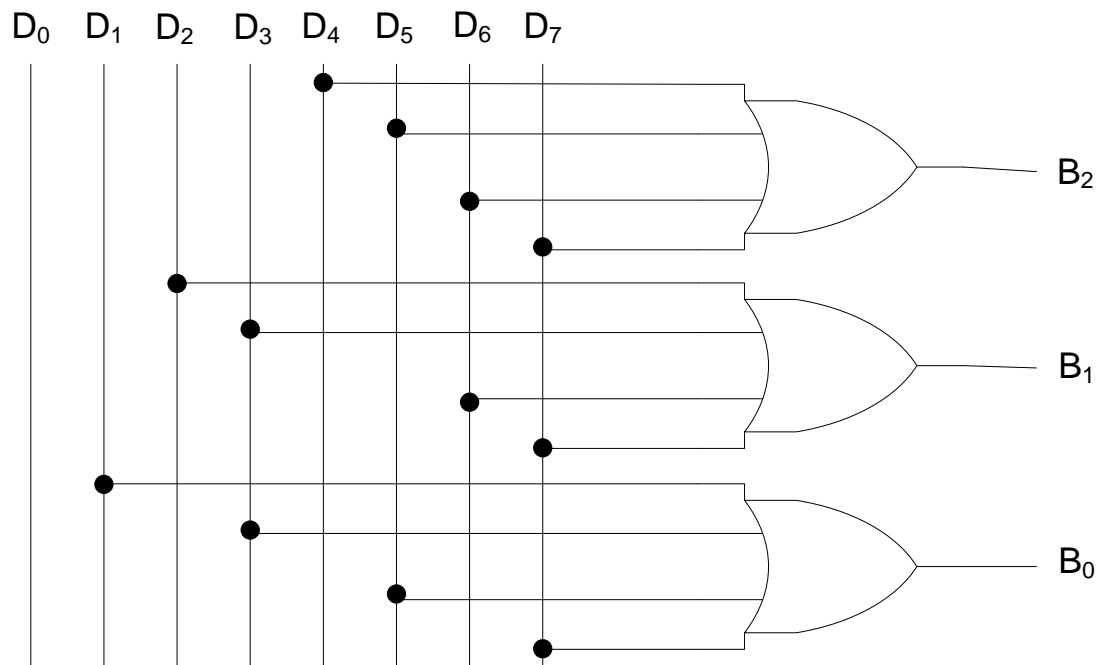
BCD	000	001	010	011	100	101	110	111
A=0	1	0	1	0	1	0	1	0
A=1	1	0	X	X	X	X	X	X
Z	1	0	1	0	1	0	1	0
Data Input	D ₀ =1	D ₁ =0	D ₂ =1	D ₃ =0	D ₄ =1	D ₅ =0	D ₆ =1	D ₇ =0



Q13. Realize a logic circuit for octal to binary encoder.

Answer: Truth table for octal to binary encoder

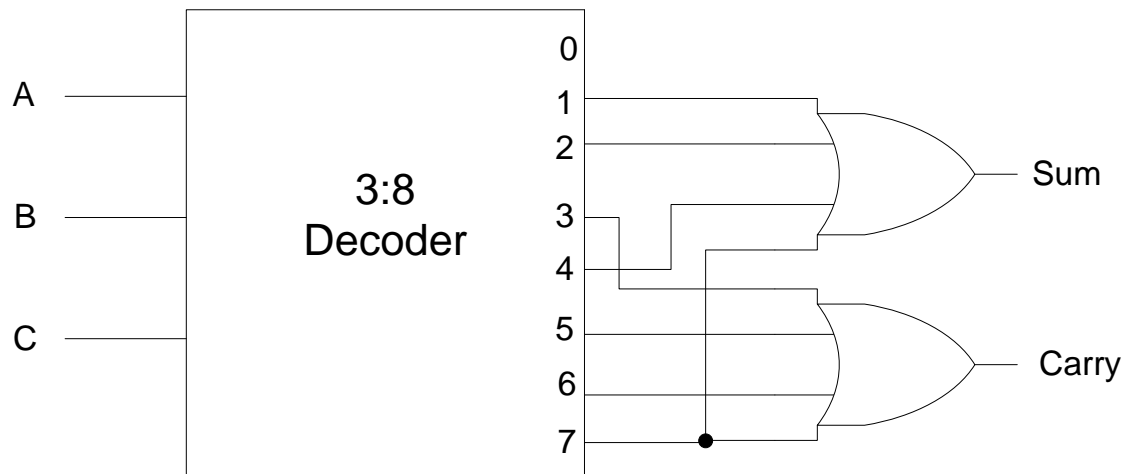
Input								Output		
D ₀	D ₁	D ₂	D ₃	D ₄	D ₅	D ₆	D ₇	B ₂	B ₁	B ₀
1	0	0	0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0	0	1
0	0	1	0	0	0	0	0	0	1	0
0	0	0	1	0	0	0	0	0	1	1
0	0	0	0	1	0	0	0	1	0	0
0	0	0	0	0	1	0	0	1	0	1
0	0	0	0	0	0	1	0	1	1	0
0	0	0	0	0	0	0	1	1	1	1



Q14. Implement a full adder using a 3 to 8 decoder.

Answer: Truth table for full adder

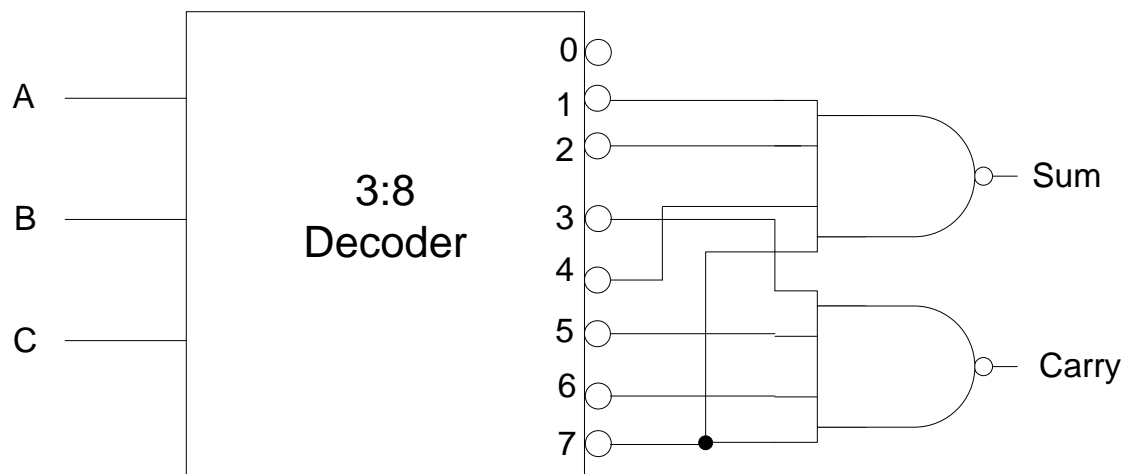
A	B	C	Sum	Carry
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1



Q15. Implement full adder using IC 74138

Answer: Truth table for full adder

A	B	C	Sum	Carry
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1



Q16. Implement 3 bit binary to gray code conversion by using IC 74139.

Answer: Truth table for converting 3 bit binary to gray code

Binary code			Gray code		
A	B	C	X	Y	Z
0	0	0	0	0	0
0	0	1	0	0	1
0	1	0	0	1	1
0	1	1	0	1	0
1	0	0	1	1	0
1	0	1	1	1	1
1	1	0	1	0	1
1	1	1	1	0	0

K-Map for X:

AB \ C	0	1
00	0	0
01	0	0
11	1	1
10	1	1

$$X=A$$

K-Map for Y:

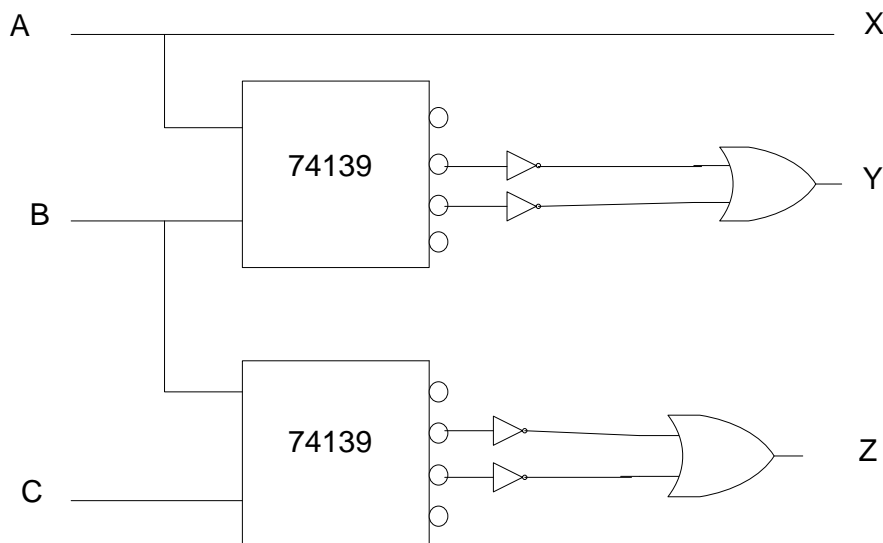
AB \ C	0	1
00	0	0
01	1	1
11	0	0
10	1	1

$$Y=\bar{A}B+AB$$

K-Map for Z:

AB \ C	0	1
00	0	1
01	1	0
11	1	0
10	0	1

$$Z=B\bar{C}+\bar{B}C$$



Q17. Design a priority encoder for a system with a 3 inputs, the middle bit with highest priority encoding to 10, the MSB with the next priority encoding to 11, while the LSB with least priority encoding to 01.

Answer: Truth table of the priority encoder

Input			Output	
A	B	C	X	Y
0	0	0	0	0
0	0	1	0	1
0	1	0	1	0
0	1	1	1	0
1	0	0	1	1
1	0	1	1	1
1	1	0	1	0
1	1	1	1	0

K-Map for X:

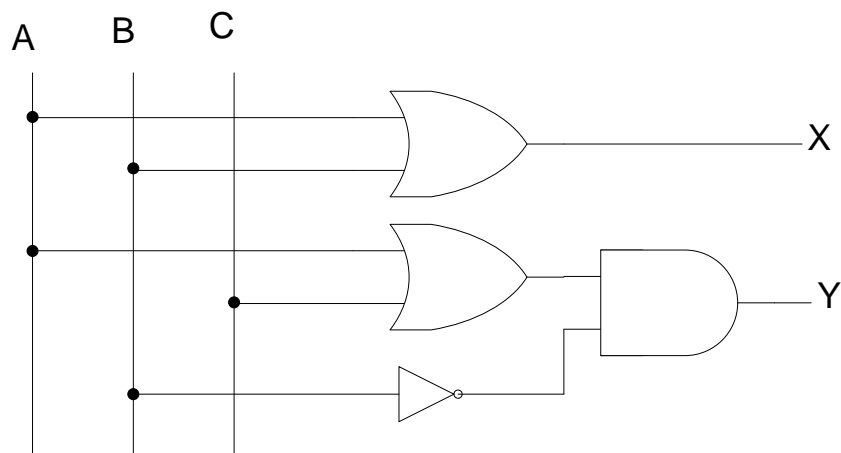
AB \ C	0	1
00	0	0
01	1	1
11	1	1
10	1	1

$$X = A + B$$

K-Map for Y:

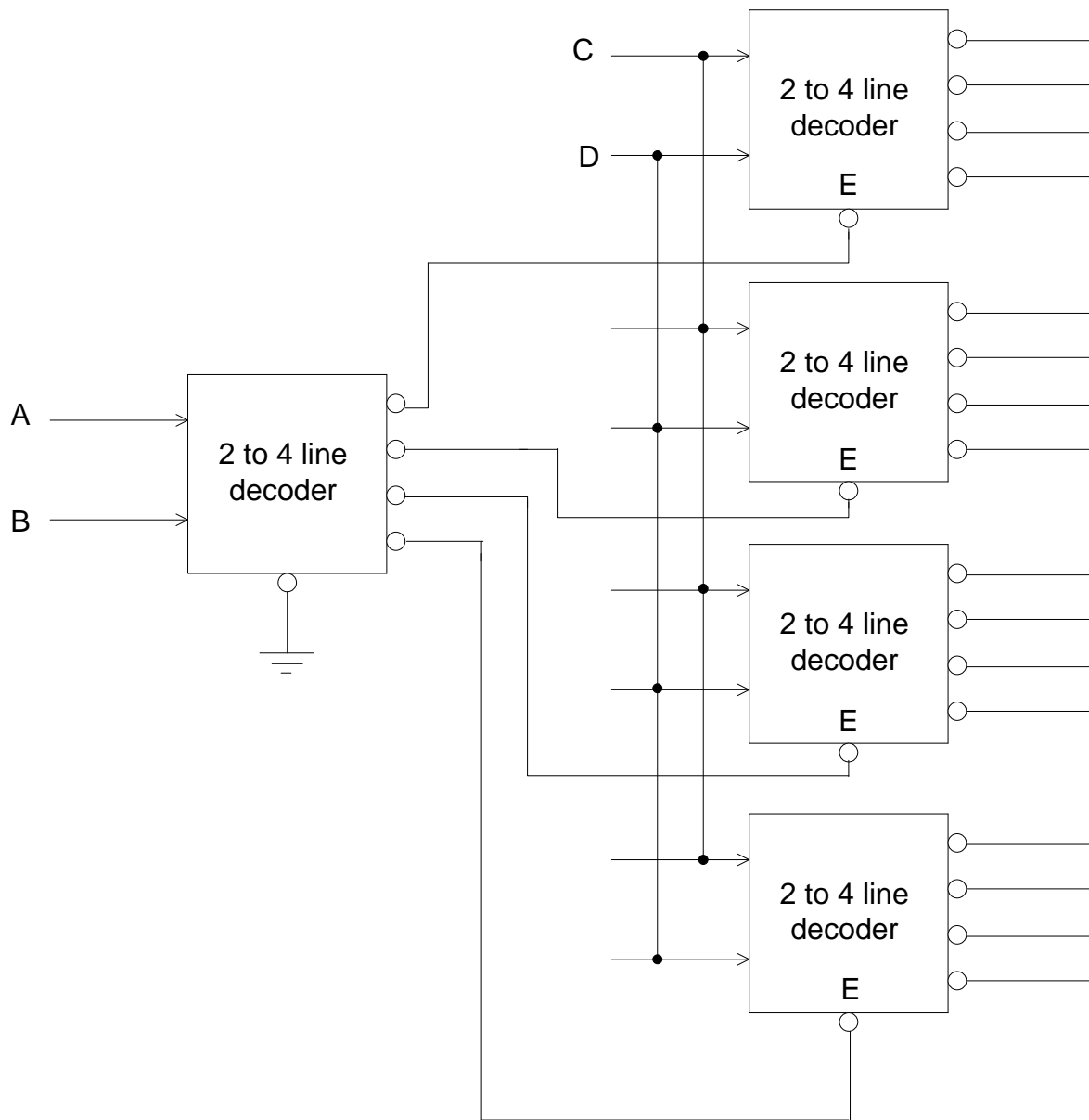
AB \ C	0	1
00	0	1
01	0	0
11	0	0
10	1	1

$$Y = \bar{A}\bar{B} + \bar{B}C = \bar{B}(A + C)$$



Q18. Design a 4 to 16 line decoder using 2 to 4 line decoder which has the active low outputs as active low enable input. Explain its operation.

Answer:



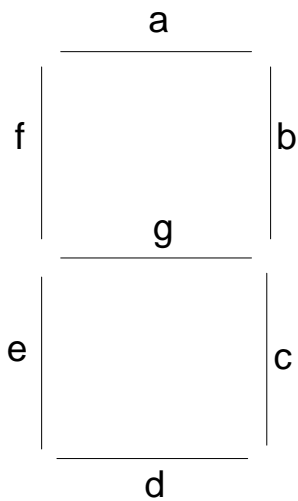
Q19. Write the comparisons between PLA and PAL.

Answer:

PAL	PLA
The output OR-gate array is fixed while the input AND gate array is fusible linked and thus programmable.	Both output OR-gate array and input AND gate array are fusible linked.
PAL is easier to program.	PLA is more complicated since the number fusible links are more compared to PAL
PAL is less expensive.	PLA is more expensive compared to PAL.

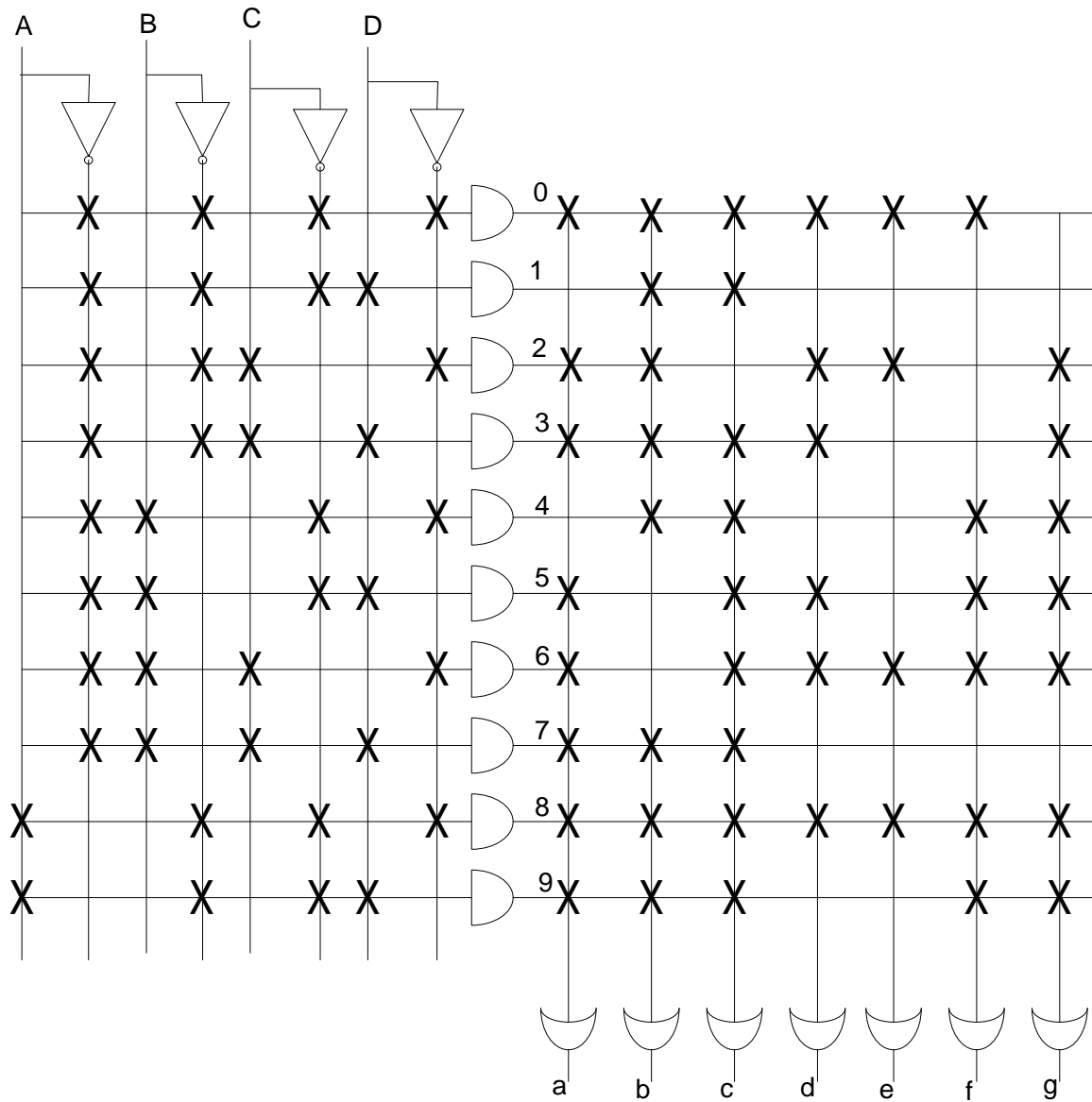
Q20. Design 7-segments decoder using PLA.

Answer: Seven segment indicator:



Following table shows the segments should light up to display a number.

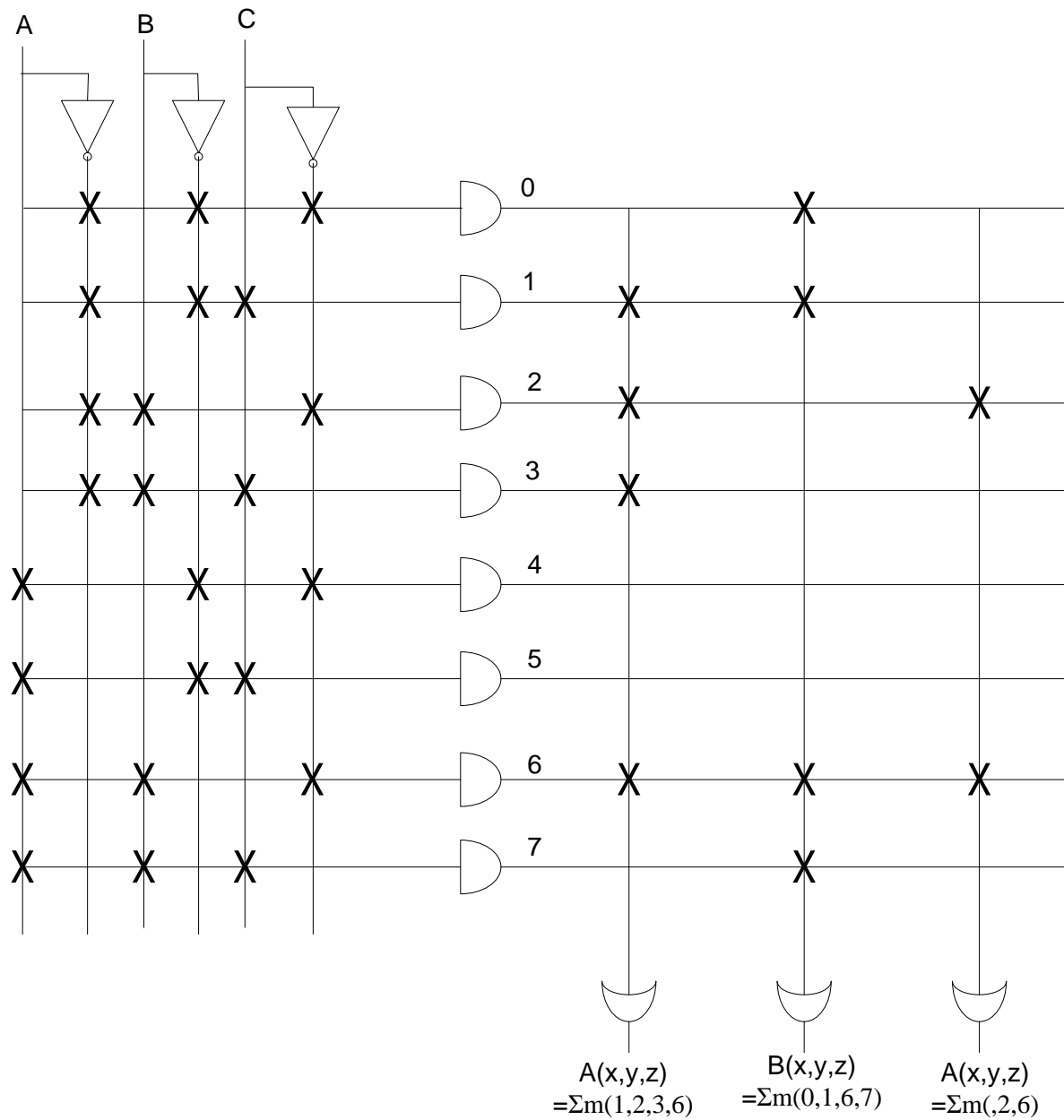
Number to display	Segments to light up
0	a,b,c,d,e,f
1	b,c
2	a,b,d,e,g
3	a,b,c,d,g
4	b,c,f,g
5	a,c,d,f
6	a,c,d,e,f,g
7	a,b,c
8	a,b,c,d,e,f,g
9	a,b,c,f,g



Q21. Implement the following function using PLA:

$$A(x,y,z)=\Sigma m(1,2,3,6); B(x,y,z)=\Sigma m(0,1,6,7); C(x,y,z)=\Sigma m(2,6)$$

Answer:



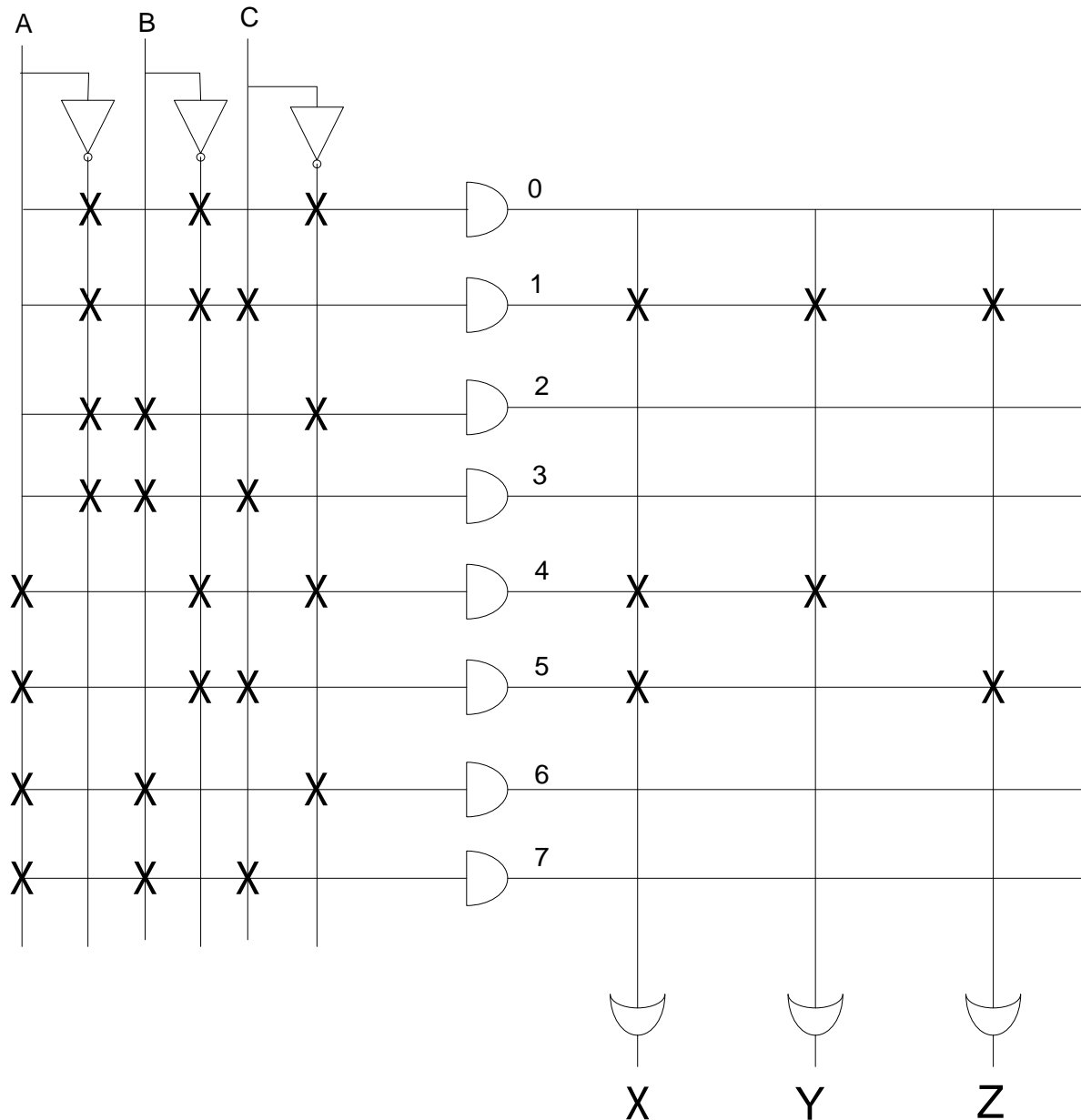
Q22. Draw the PLA circuit and realize the Boolean functions:

$$X = A'B'C + AB'C' + B'C, Y = A'B'C + AB'C', Z = B'C$$

$$\text{Answer: } X = A'B'C + AB'C' + B'C = A'B'C + AB'C' + B'C(A + A') = A'B'C + AB'C' + AB'C$$

$$Y = A'B'C + AB'C'$$

$$Z = B'C = B'C(A + A') = AB'C + A'B'C$$

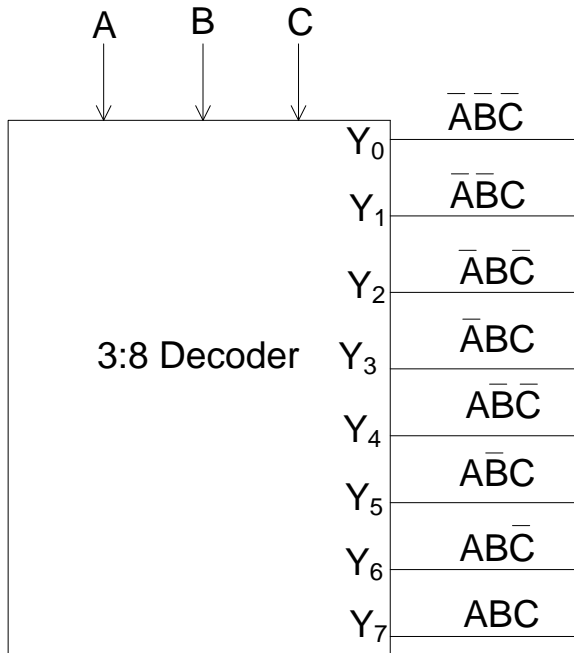


Q23. Describe the working principle of 3:8 decoder. Design a circuit that realizes the following functions using a 3:8 decoder and multi input OR gates.

$$(i) F_1(A, B, C) = \sum m(1, 3, 7) \quad (ii) F_2(A, B, C) = \sum m(2, 3, 5)$$

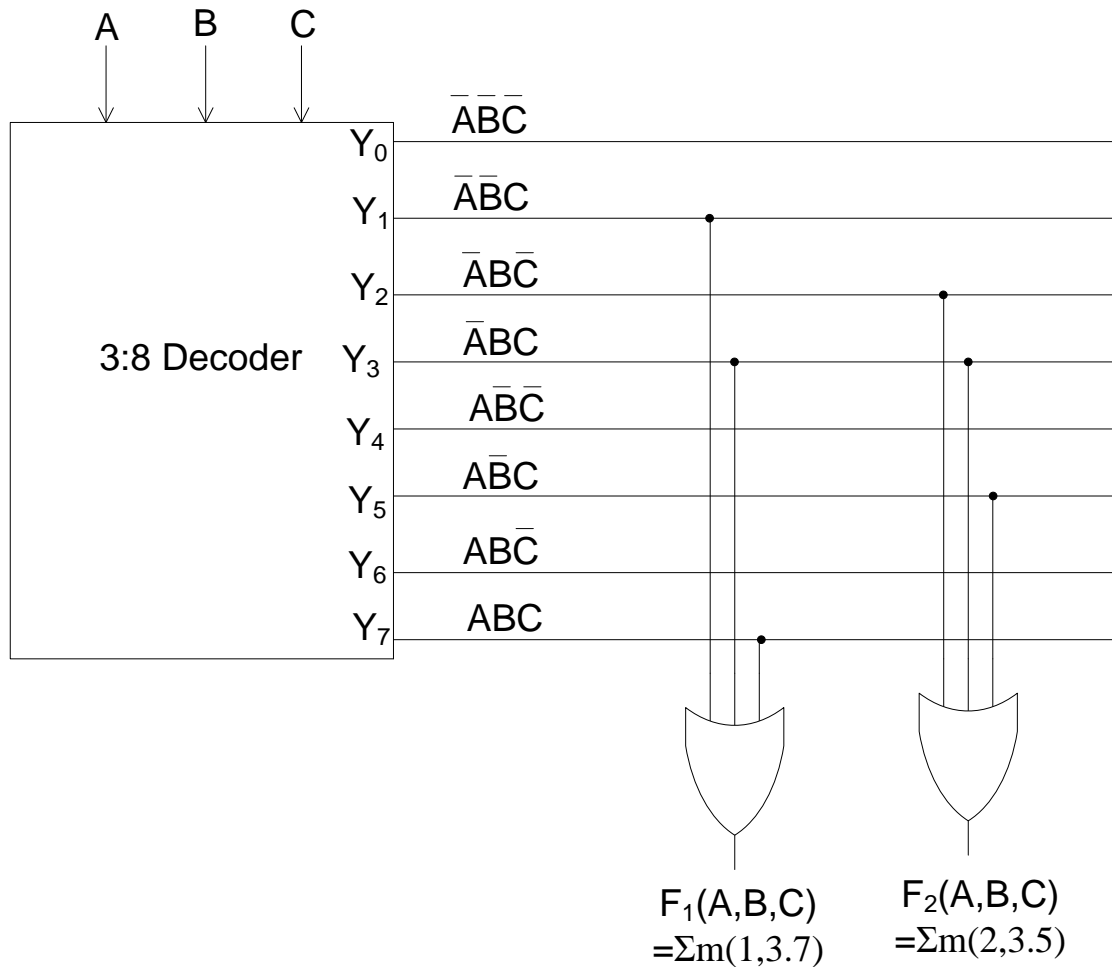
Answer: Working principle of 3:8 decoder:

There are 3 inputs and 8 outputs in a 3:8 decoder. One of the output is HIGH and remaining seven are LOW according to inputs. This is shown in truth table.



Truth table of 3:8 Decoder :

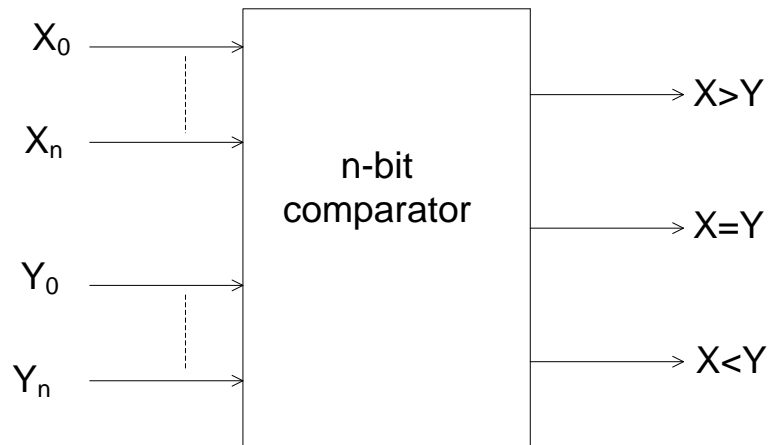
A	B	C	Y ₀	Y ₁	Y ₂	Y ₃	Y ₄	Y ₅	Y ₆	Y ₇
0	0	0	1	0	0	0	0	0	0	0
0	0	1	0	1	0	0	0	0	0	0
0	1	0	0	0	1	0	0	0	0	0
0	1	1	0	0	0	1	0	0	0	0
1	0	0	0	0	0	0	1	0	0	0
1	0	1	0	0	0	0	0	1	0	0
1	1	0	0	0	0	0	0	0	1	0
1	1	1	0	0	0	0	0	0	0	1



Q24. What is magnitude comparator? Design one bit comparator and write the truth table, logic circuit using basic gates.

Answer: A magnitude comparator compares two binary numbers and it produces an output showing the comparison of the two input numbers.

For example, two n-bit binary numbers $X = X_0X_1...X_n$ and $Y = Y_0Y_1...Y_n$ are compared. There are three outputs. The outputs are for $X > Y$, $X = Y$ and $X < Y$ as shown in the figure below.



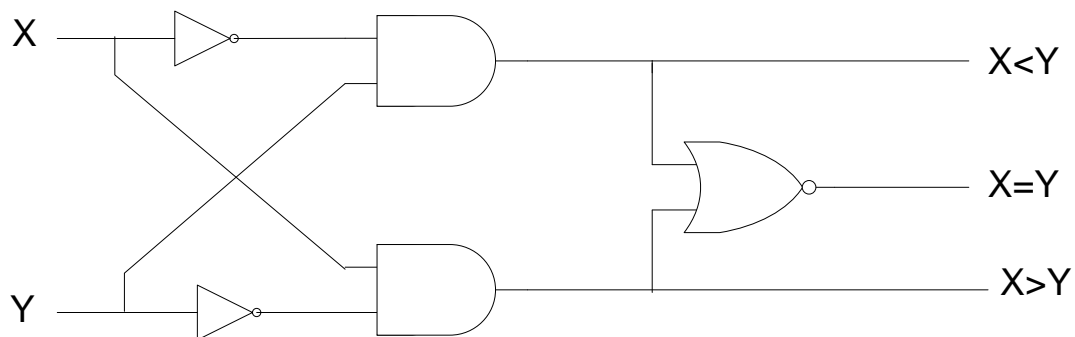
Designing of one bit comparator:

Truth table

Input		Output		
X	Y	X>Y	X=Y	X<Y
0	0	0	1	0
0	1	0	0	1
1	0	1	0	0
1	1	0	1	0

If G, L, E stand for greater than, less than and equal to respectively, then

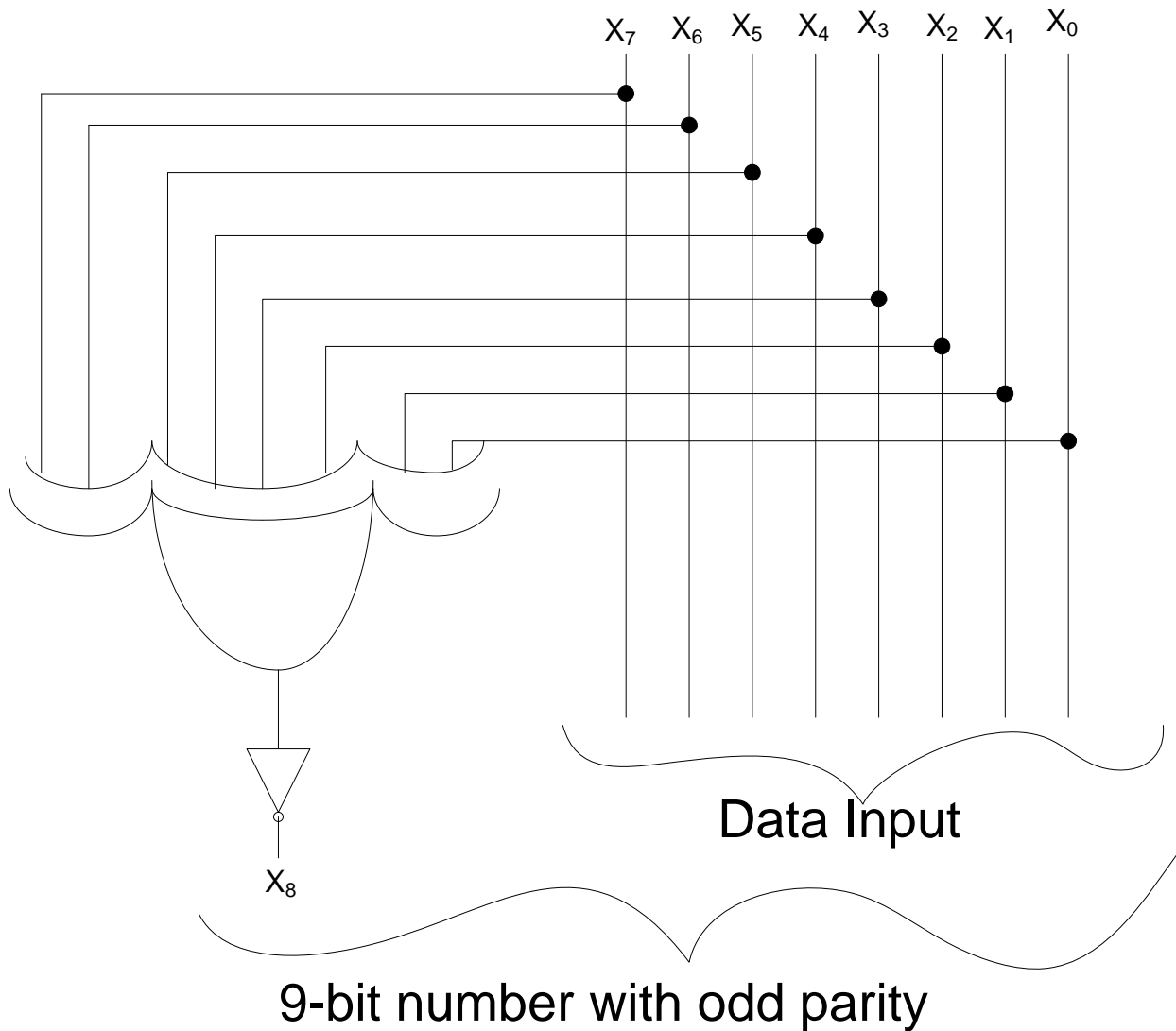
$(X > Y): G = XY'$; $(X < Y): L = X'Y$; $(X = Y): E = X'Y' + XY = (XY' + X'Y)' = (G + L)'$



Q25. What is parity generator? Explain with an example.

Answer: A parity generator is a logic circuit which produces either even parity number or odd parity number as per requirement.

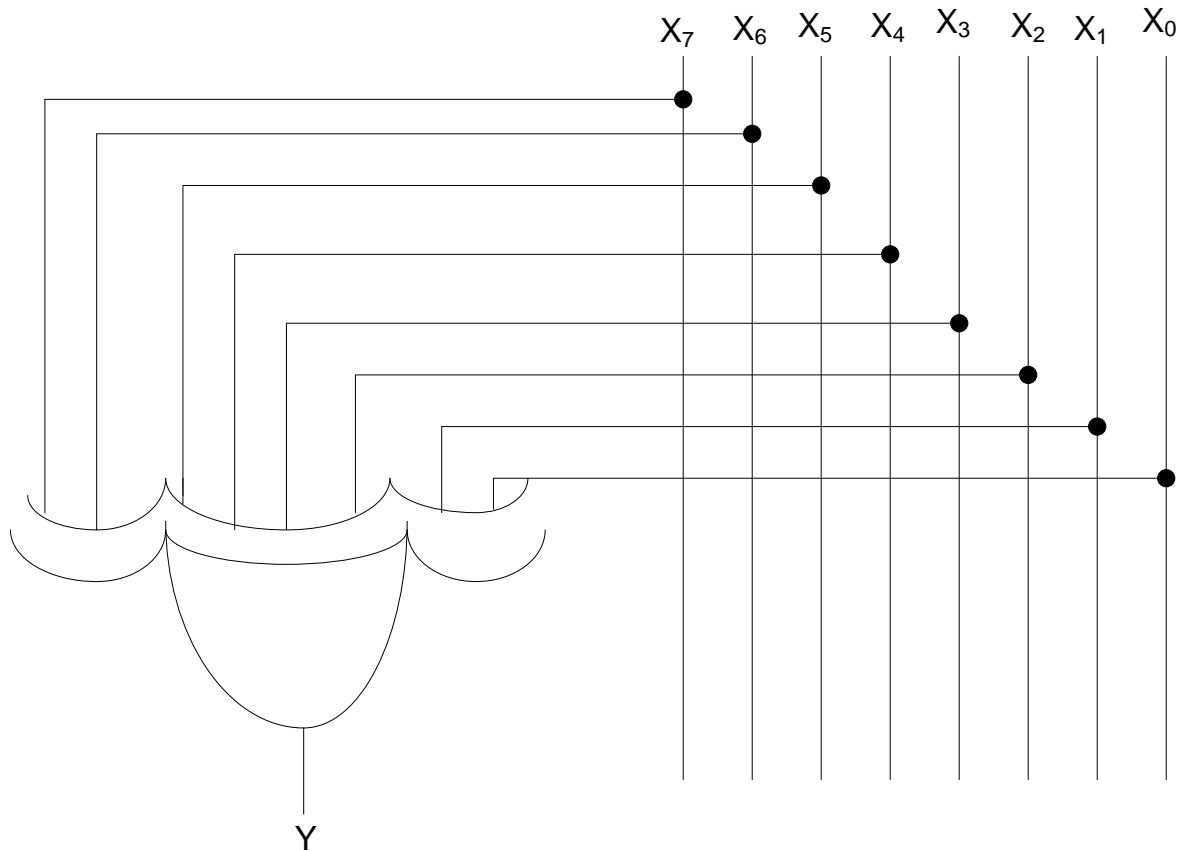
For example:



Q26. What is parity checker? Explain with example.

Answer: A parity checker check the parity of a number whether the number is of even parity or odd parity.

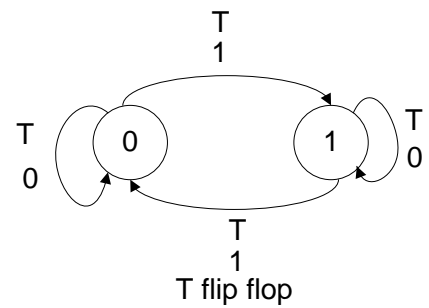
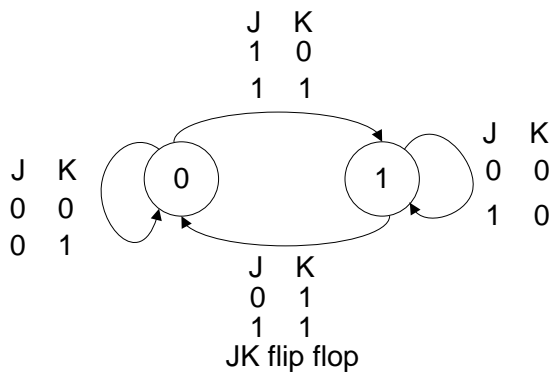
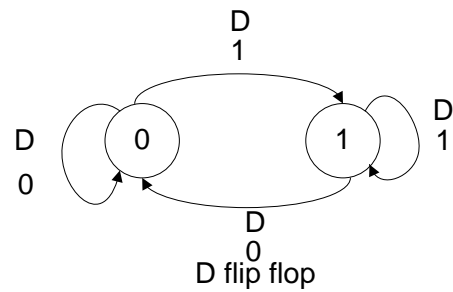
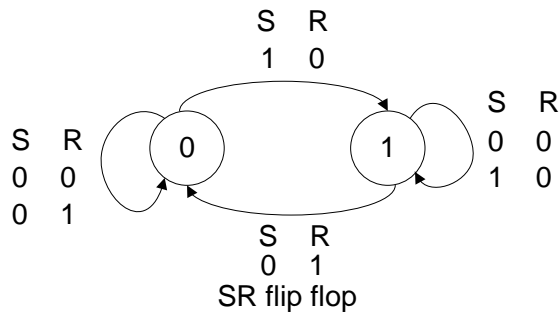
For example: The exclusive OR gate produces an output 1 when the input ($X_0 \dots X_7$) is of odd parity and produces 0 when the input is of even parity.



Q27. Give state transition diagram of SR, D, JK and T flip flops.

Answer:

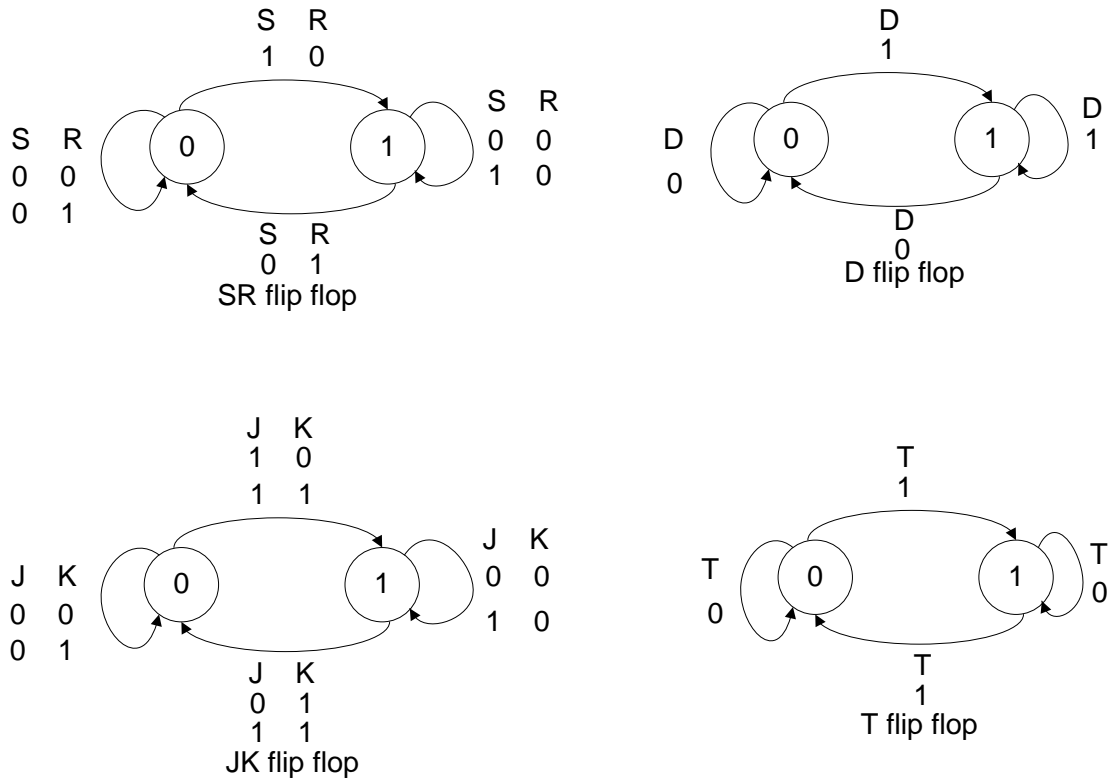
State Transition Diagram



Q28. Obtain the characteristic equation of SR, JK, D and T flip flops.

Answer:

State Transition Diagram



Excitation Table for SR, JK, D and T flip flop is given below is prepared State Transition Diagram above

$Q_n \rightarrow Q_{n+1}$	S	R	J	K	D	T
0 → 0	0	X	0	X	0	0
0 → 1	1	0	1	X	1	1
1 → 0	0	1	X	1	0	1
1 → 1	X	0	X	0	1	0

From the Excitation Table, K-map is formed and then the characteristic equation is determined.

Q _n \ SR				
	00	01	11	10
0	0	0	x	1
1	1	0	x	1

Characteristic Equation for SR flip flop is

$$Q_{n+1} = S + \bar{R}Q_n$$

Q _n \ JK				
	00	01	11	10
0	0	0	1	1
1	1	0	0	1

Characteristic Equation for JK flip flop is

$$Q_{n+1} = J\bar{Q}_n + \bar{K}Q_n$$

Q _n \ D		
	0	1
0	0	1
1	0	1

Characteristic Equation for D flip flop is

$$Q_{n+1} = D$$

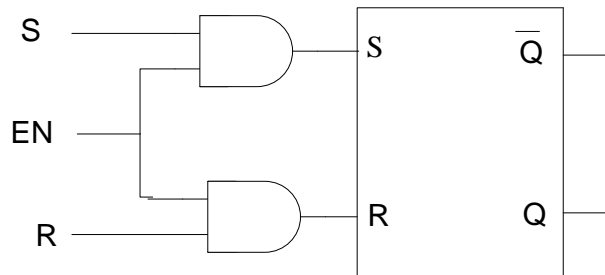
Q _n \ T		
	0	1
0	0	1
1	1	0

Characteristic Equation for T flip flop is

$$Q_{n+1} = T\bar{Q}_n + \bar{T}Q_n$$

Q29. Explain the operation of a gated SR latch with a logic diagram and truth table.

Logic diagram and truth table of gated SR flip flop is shown below:



Logic Diagram

EN	S	R	Q_{n+1}
1	0	0	Q_n (No Change)
1	0	1	0
1	1	0	1
1	1	1	Illegal
0	x	x	Q_n (No Change)

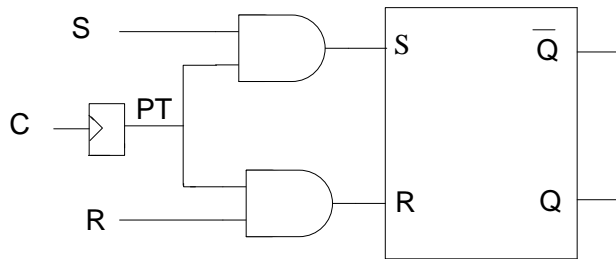
Truth table

When the Enable (EN) input is high, information at the R and S inputs will be transmitted directly to the outputs. The latch is said to be enabled. When the Enable (EN) input is low, the outputs of the AND gates are low and information at the R and S inputs will not be transmitted to the outputs. The latch is said to be disabled.

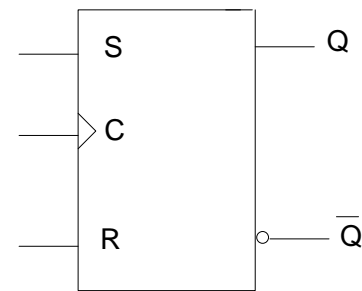
It is possible to strobe or clock the flip flop in order to store information at any time and then hold the stored information for any desired period of time. This flip flop is called a gated or clocked RS flip flop.

Q30. Explain the operation of edge triggered 'SR' flip flop with the help of a logic diagram and truth table. Also draw the relevant waveforms.

Answer: Positive edge triggered 'SR' flip flop



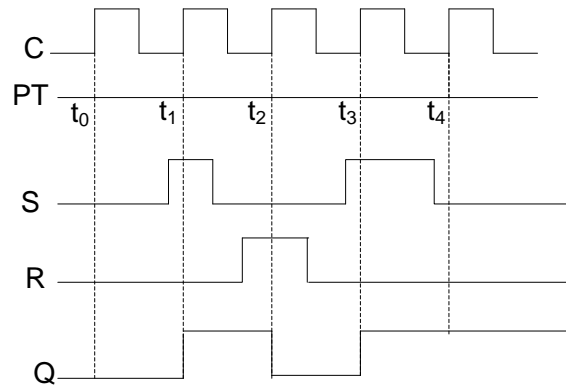
Logic Diagram



IEEE Symbol

C	S	R	Q_{n+1}
↑	0	0	Q_n (No Change)
↑	0	1	0 (Reset)
↑	1	0	1 (Set)
↑	1	1	Illegal

Truth table



Waveform of positive edge triggered RS flip flop

Positive edges occur at t_0, t_1, t_2, t_3 and t_4 .

At t_0 , $S=0$ and $R=0$, hence no change in the output and $Q=0$.

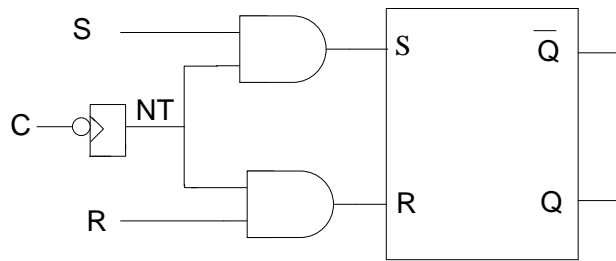
At t_1 , $S=1$ and $R=0$, hence the output is set and $Q=1$.

At t_2 , $S=0$ and $R=1$, hence the output is reset and $Q=0$.

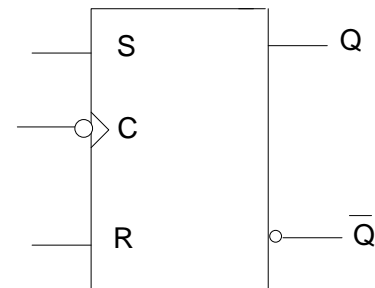
At t_3 , $S=1$ and $R=0$, hence the output is set and $Q=1$.

At t_4 , $S=0$ and $R=0$, hence no change in the output and $Q=1$.

Negative edge triggered 'SR' flip flop



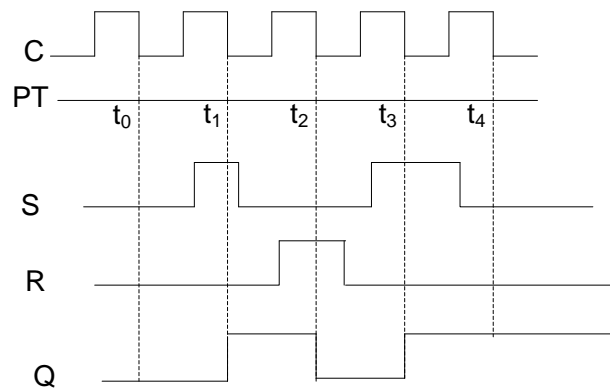
Logic Diagram



SR flip flop Symbol

C	S	R	Q_{n+1}
↓	0	0	Q_n (No Change)
↓	0	1	0 (Reset)
↓	1	0	1 (Set)
↓	1	1	Illegal

Truth table



Waveform of negative edge triggered SR flip flop

Negative edges occur at t_0, t_1, t_2, t_3 and t_4 .

At t_0 , $S=0$ and $R=0$, hence no change in the output and $Q=0$.

At t_1 , $S=1$ and $R=0$, hence the output is set and $Q=1$.

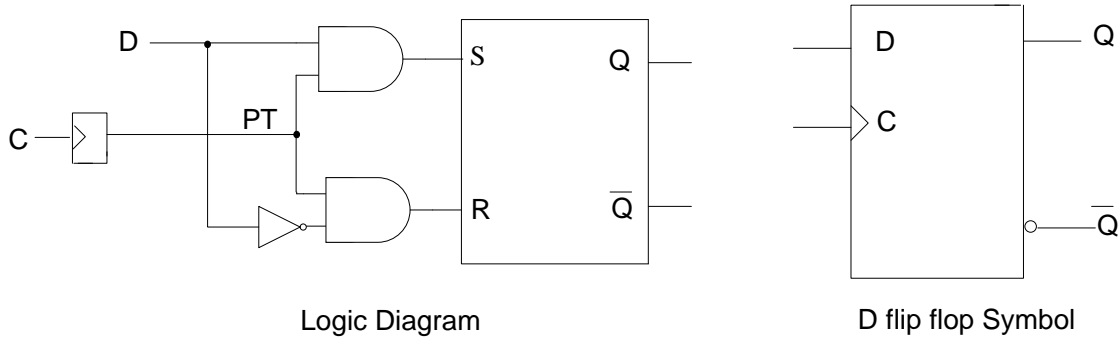
At t_2 , $S=0$ and $R=1$, hence the output is reset and $Q=0$.

At t_3 , $S=1$ and $R=0$, hence the output is set and $Q=1$.

At t_4 , $S=0$ and $R=0$, hence no change in the output and $Q=1$.

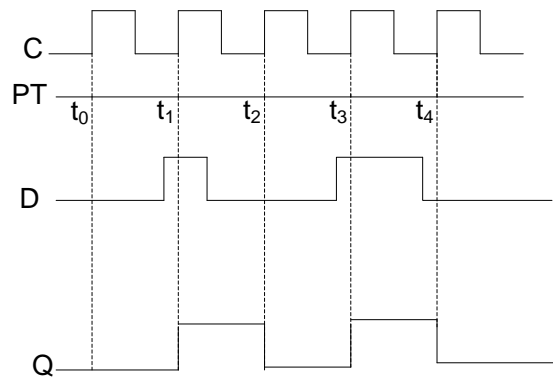
Q31. Explain the operation of edge triggered 'D' flip flop with the help of a logic diagram and truth table. Also draw the relevant waveforms.

Answer: Positive edge triggered 'D' flip flop



C	D	Q_{n+1}
0	x	Q_n (No Change)
↑	0	0
↑	1	1

Truth table



Waveform of positive edge triggered D flip flop

Positive edges occur at t_0, t_1, t_2, t_3 and t_4 .

At t_0 , $D=0$, hence the output is low and $Q=0$.

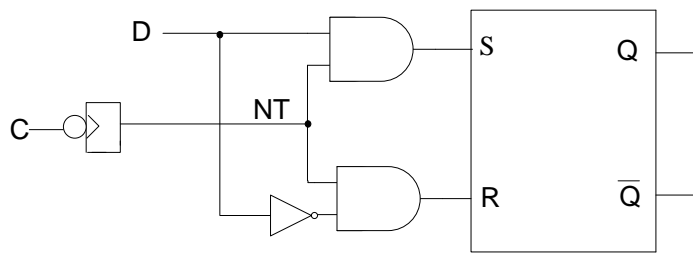
At t_1 , $D=1$, hence the output is high and $Q=1$.

At t_2 , $D=0$, hence the output is low and $Q=0$.

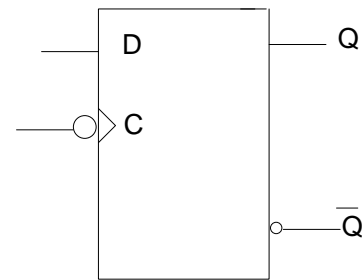
At t_3 , $D=1$, hence the output is high and $Q=1$.

At t_4 , $D=0$, hence no change in the output and $Q=1$.

Negative triggered D flip flop



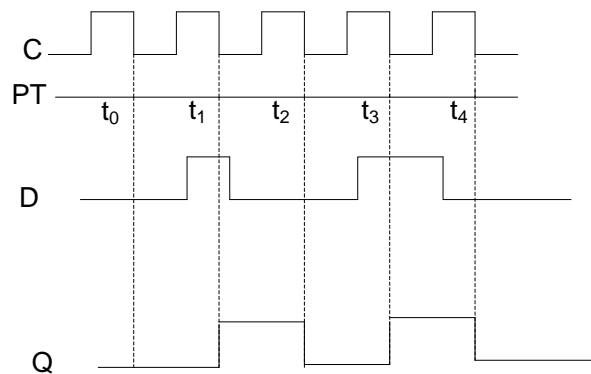
Logic Diagram



D flip flop Symbol

C	D	Q_{n+1}
0	x	Q_n (No Change)
↓	0	0
↓	1	1

Truth table



Waveform of positive edge triggered D flip flop

Negative edges occur at t_0, t_1, t_2, t_3 and t_4 .

At t_0 , $D=0$, hence the output is low and $Q=0$.

At t_1 , $D=1$, hence the output is high and $Q=1$.

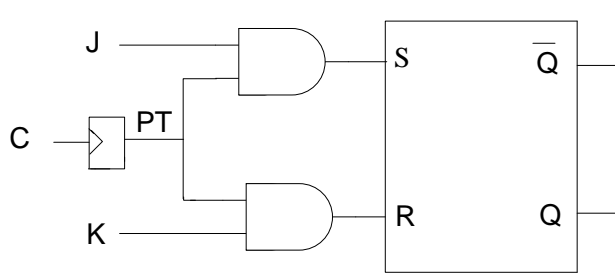
At t_2 , $D=0$, hence the output is low and $Q=0$.

At t_3 , $D=1$, hence the output is high and $Q=1$.

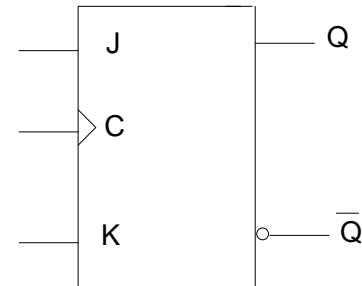
At t_4 , $D=0$, hence no change in the output and $Q=1$.

Q32. Explain the working of pulse triggered JK flip flop with typical JK flip flop waveform.

Answer: Positive Edge Triggered JK flip flop



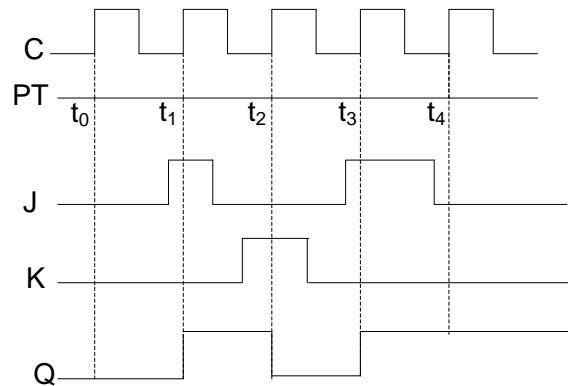
Logic Diagram



JK flip flop Symbol

C	J	K	Q_{n+1}
↑	0	0	Q_n (No Change)
↑	0	1	0
↑	1	0	1
↑	1	1	Toggle

Truth table



Waveform of positive edge triggered JK flip flop

Positive edges occur at t_0, t_1, t_2, t_3 and t_4 .

At t_0 , $J=0$ and $K=0$, hence no change in the output and $Q=0$.

At t_1 , $J=1$ and $K=0$, hence the output is high and $Q=1$.

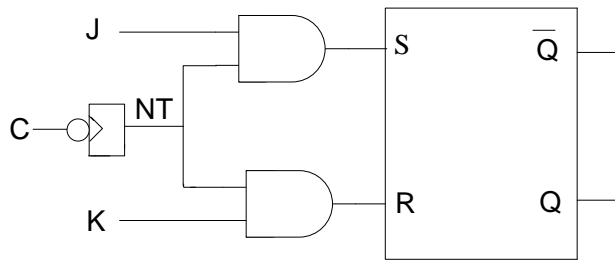
At t_2 , $J=0$ and $K=1$, hence the output is low and $Q=0$.

At t_3 , $J=1$ and $K=0$, hence the output is high and $Q=1$.

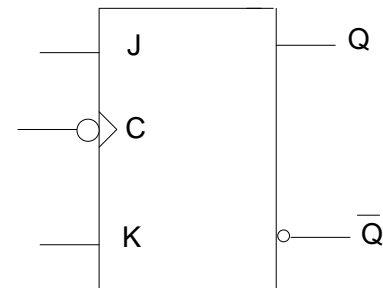
At t_4 , $J=0$ and $K=0$, hence no change in the output and $Q=1$.

:

Negative Edge Triggered JK flip flop



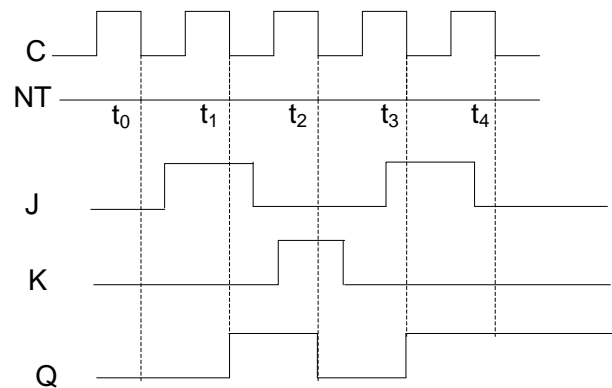
Logic Diagram



JK flip flop Symbol

C	J	K	Q_{n+1}
↓	0	0	Q_n (No Change)
↓	0	1	0
↓	1	0	1
↓	1	1	Toggle

Truth table



Waveform of positive edge triggered JK flip flop

Negative edges occur at t_0, t_1, t_2, t_3 and t_4 .

At t_0 , $J=0$ and $K=0$, hence no change in the output and $Q=0$.

At t_1 , $J=1$ and $K=0$, hence the output is high and $Q=1$.

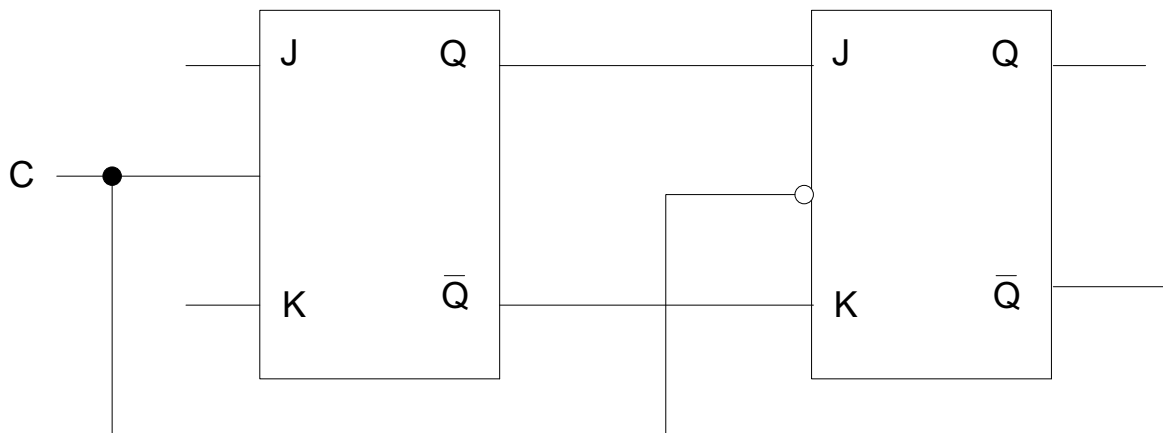
At t_2 , $J=0$ and $K=1$, hence the output is low and $Q=0$.

At t_3 , $J=1$ and $K=0$, hence the output is high and $Q=1$.

At t_4 , $J=0$ and $K=0$, hence no change in the output and $Q=1$.

Q33. Explain the working of Master Slave J K flip flops with logic diagram.

Answer:



Master Slave flip flop

Master is positive-level-triggered and the slave is negative-level-triggered. The master responds to its J and K inputs before the slave.

If $J=1$ and $K=0$, the master sets on the positive clock transition. The high Q output of the master drives the J input of the slave. So, on the negative clock transition, the slave sets, thus copying the action of the master.

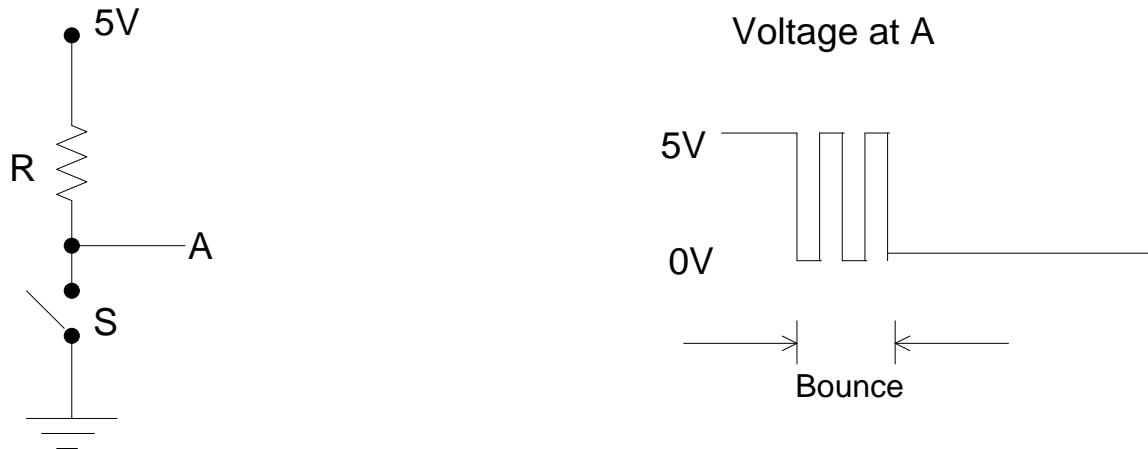
If $J=0$ and $K=1$, the master resets on the positive clock transition. The high \bar{Q} output of the master drives the K input of the slave. So, on the negative clock transition, the slave resets, thus copying the action of the master.

If $J=1$ and $K=1$, the master toggle on the positive clock transition. The slave also toggle at the negative clock transition thus copying the action of the master.

If $J=0$ and $K=0$, the master and the slave both are disabled, thus copying the action of the master.

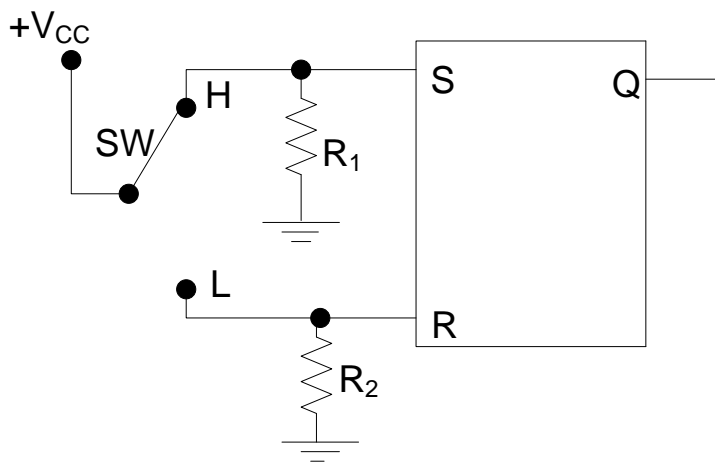
Q34. What is contact bounce? With neat diagram, explain the working principles of Switch De bounce circuit.

Answer:



Any mechanical switching device consists of a moving contact arm restrained by some spring system. As a result, when a mechanical switch is closed, the arm is moved from one stable position to other and the arm bounces much as a hard ball bounces when dropped on a hard surface. This phenomenon is known as contact bounce. When switch S is closed, due to contact bounce the voltage at the A is shown in the above figure.

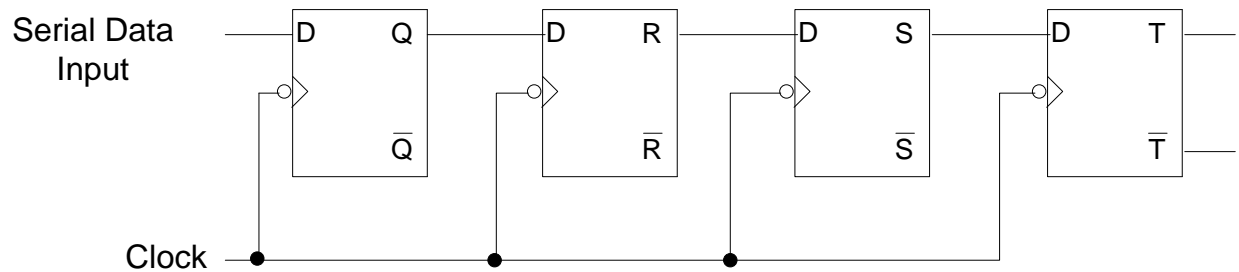
RS Latch Debounce Circuit



When switch(SW) is moved to the position H, $R=0$ and $S=1$. Bouncing occurs at S due to contact bounce of the switch. The flip flop treat as high and low inputs. The flip flop will be set with $Q=1$ at the first high of the contact bounce. When the switch continue to bounce, losing contact, the input signals are $R=S=0$, thus the flip flop remains at $Q=1$. As a result, the flip flop responds only to the first high of the contact bounce.

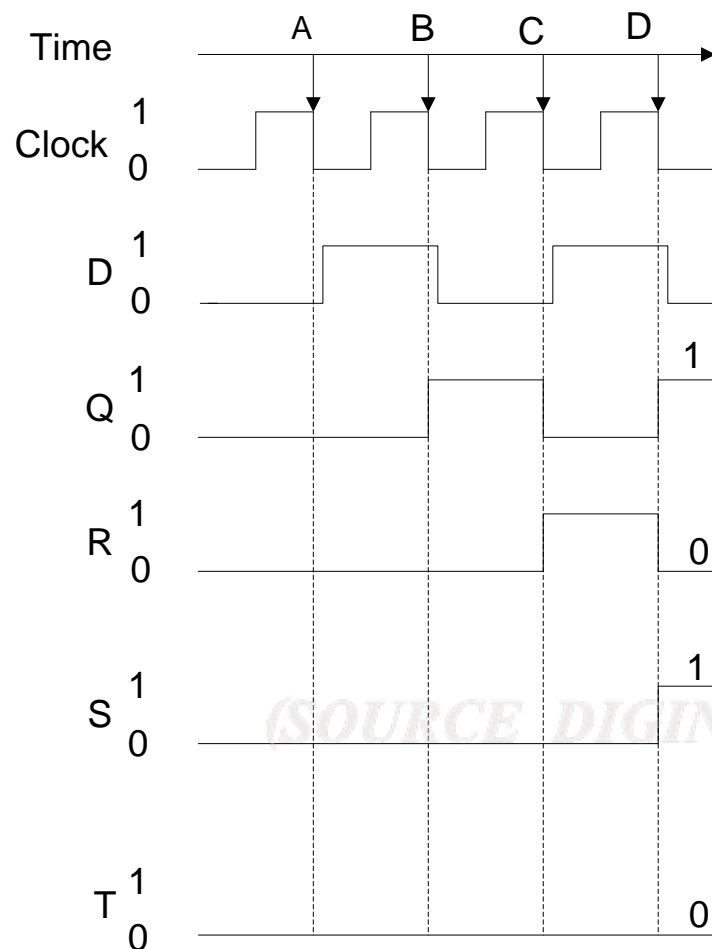
Q1. Explain 4-bit Serial in-Serial out register.

Answer: 4-bit serial in-serial out register is shown below:



The waveform for the above circuit is shown below:

It has been assumed that initially $Q=0$, $R=0$, $S=0$ and $T=0$.



At clock edge A: $DQRS=0000$. As the clock trigger at A, the values at $DQRS$ is transferred to $QRST$ and $QRST=0000$.

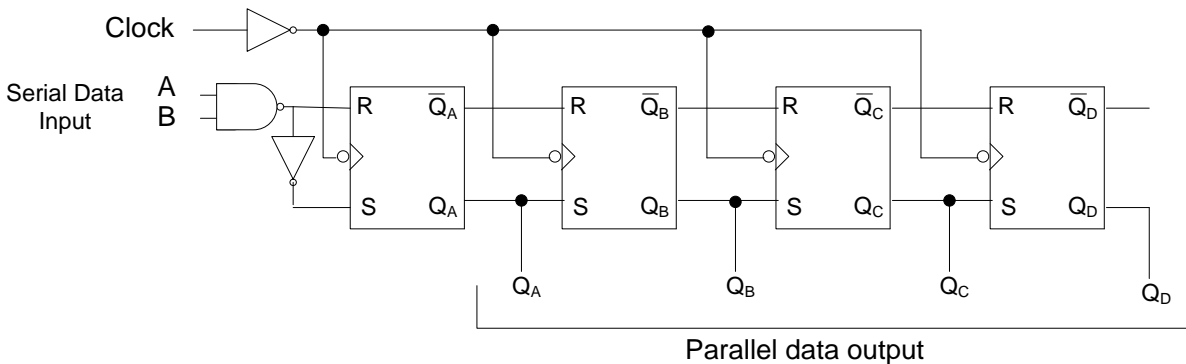
At clock edge B: DQRS=1000. As the clock trigger at B, the values at DQRS is transferred to QRST and QRST=1000.

At clock edge C: DQRS=0100. As the clock trigger at C, the values at DQRS is transferred to QRST and QRST=0100.

At clock edge D: DQRS=1010. As the clock trigger at B, the values at DQRS is transferred to QRST and QRST=1010.

Q2. Explain 4-bit Serial in-parallel out register.

Answer: 4-bit serial in-parallel out register is shown below:



Data shifted in serially, but shifted out in parallel as shown in the figure above. In order to shift the data out in parallel, output of each flip flop is connected to output pin.

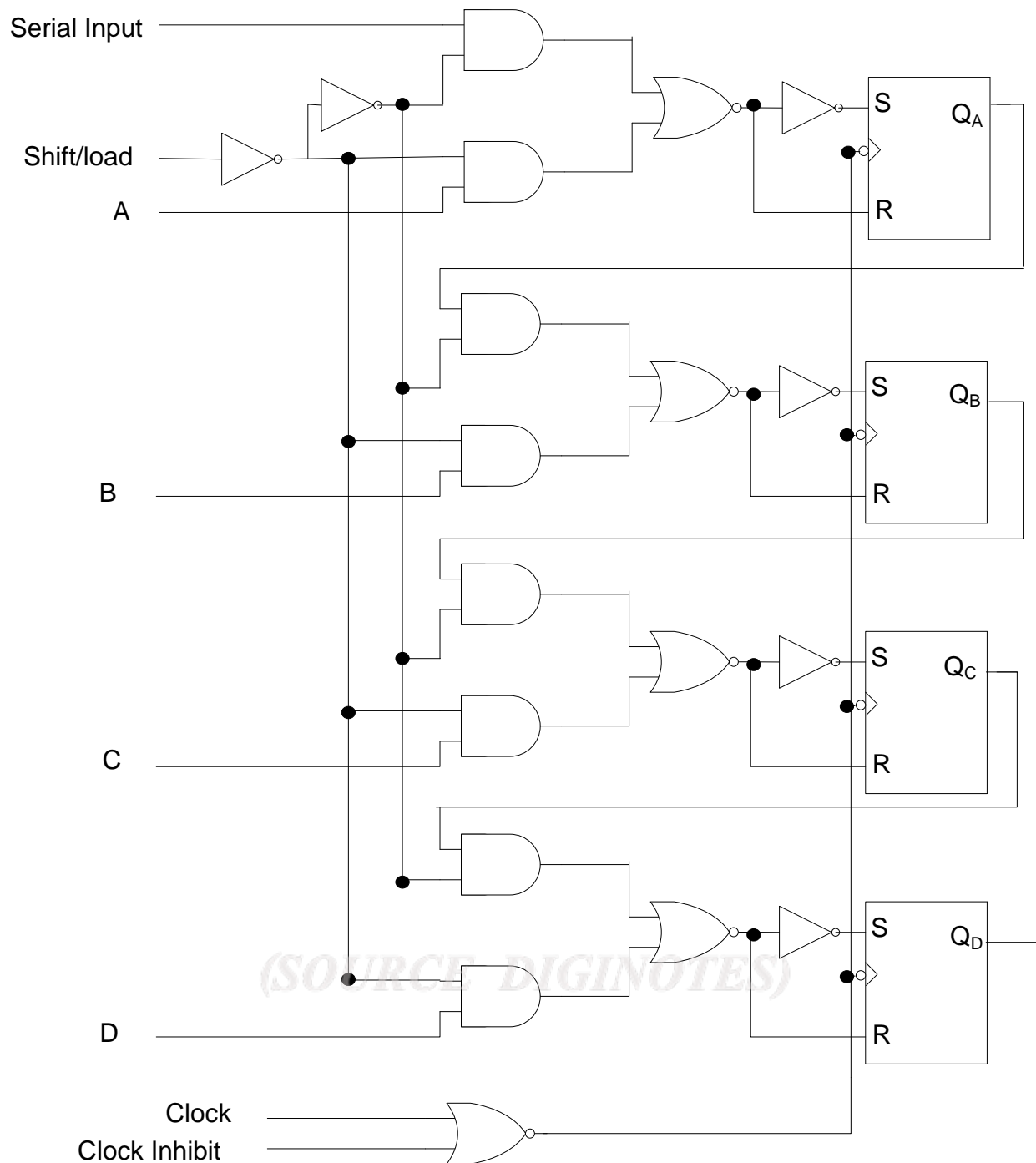
Suppose that the serial data is connected to A, and then B can be used as a control line.

If B is held high, then the NAND gate is enabled and the serial input data passes through the NAND gate is inverted. The input data is shifted serially into the register.

If B is held low, then the NAND gate output is high irrespective of input data.

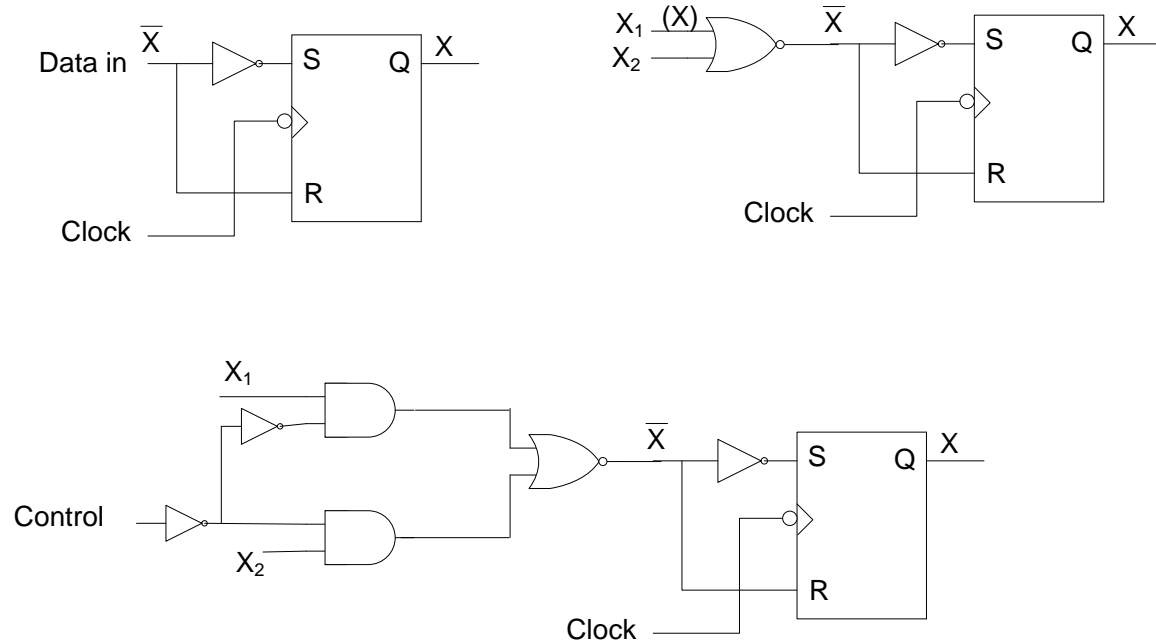
Q3. Explain 4-bit parallel in-Serial out register.

Answer: 4-bit parallel in serial out is shown below.



The above logic block diagram shows 4-bit parallel in (A, B, C and D) and serial out register. This can also be used as serial in if data is entered at Serial Input terminal as shown.

Analysis of the above circuit is given below:



The clocked RS flip flop and the attached inverter form a type D flip flop. If a data bit X is to be clocked into the, the complement of X must be present at the input.

If one leg of the NOR gate is at ground level, a data bit X at the other leg is inverted. This NOR gate provide option of entering data from two different sources, either X_1 or X_2 .

Addition of two AND gates and two inverters allow the selection of data selection of data X_1 or data X_2 .

If the control line is high, the upper AND gate is enabled and lower AND gate is disabled.

If the control line is low, upper AND gate is disabled and the lower AND gate is enabled.

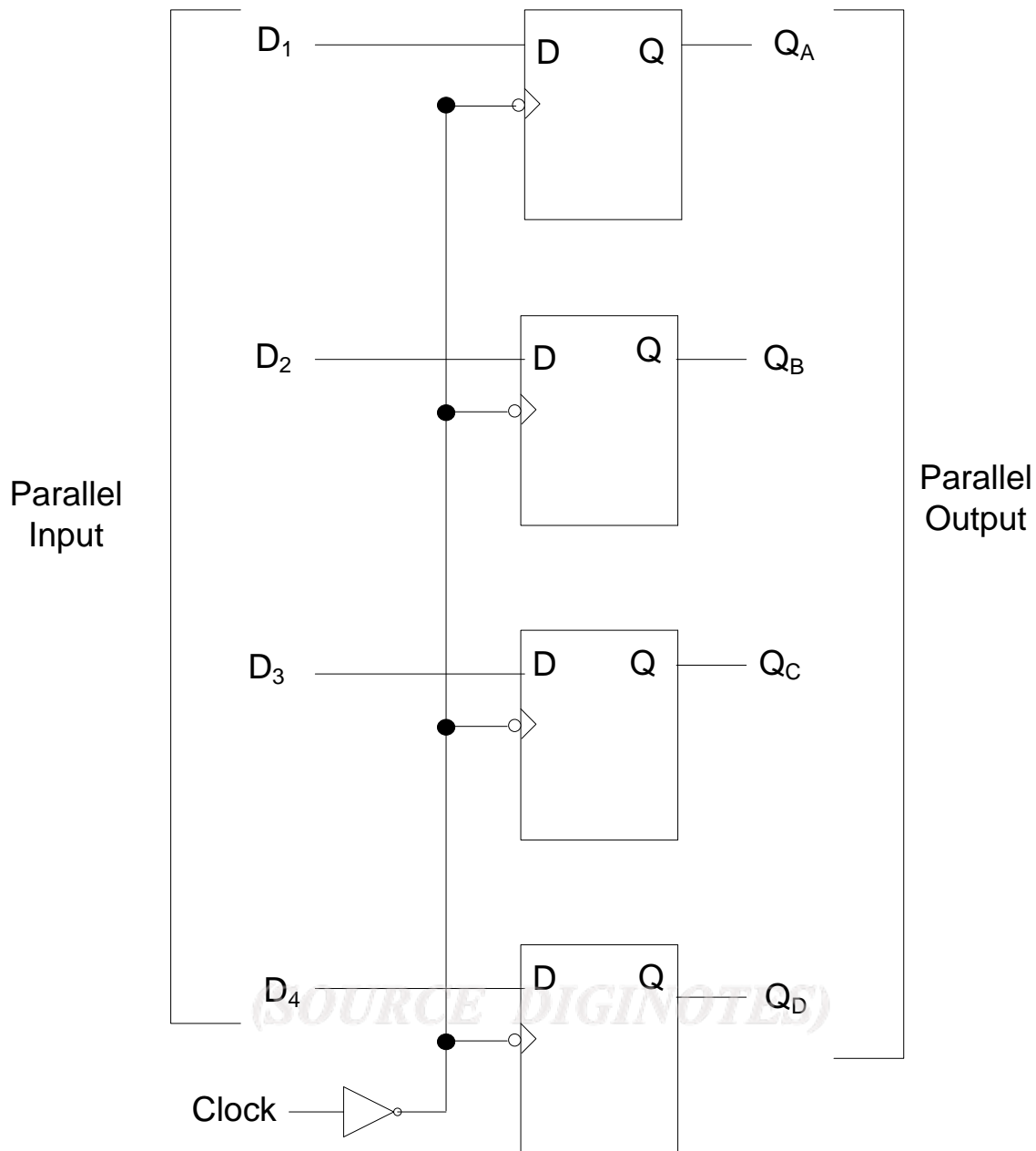
Control line is high: Data bit at X_1 will be shifted into the flip flop at the next clock pulse.

Control line is low: Data bit at X_2 will be shifted into the flip flop at the next clock pulse.

Shift/Load is low: A single clock transition load data into the register in parallel.

Q4. Explain 4-bit parallel in-parallel out register.

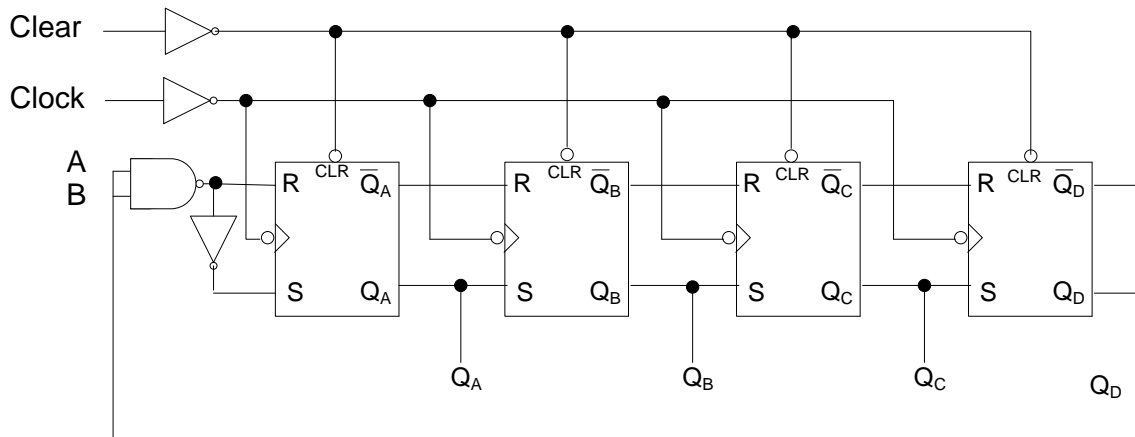
Answer: 4-bit parallel in parallel out is shown below.



Data D_1 through D_4 are shifted into the register with clock pulse. The stored data is immediately available in parallel at the output Q_1 through Q_4 . This type of register is used to store data is called data latch or data register..

Q5. Explain 4-bit Ring Counter.

Answer: Following is the 4-bit ring counter.



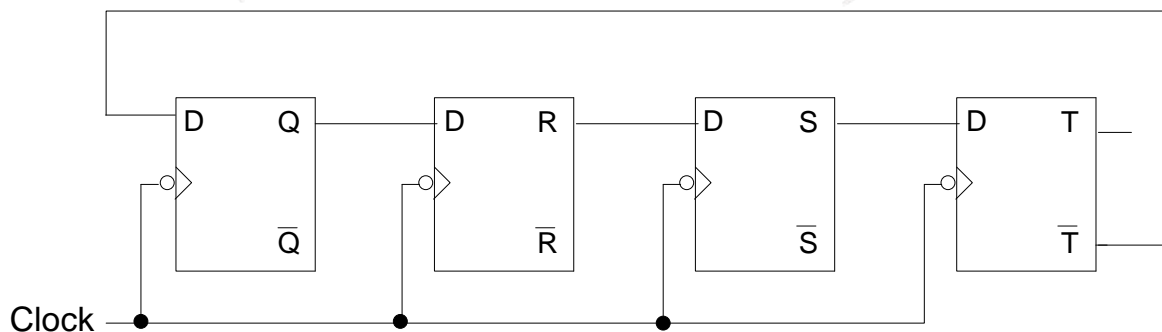
Output of the last flip flop Q_D is feedback to the input of the first flip flop.

State table of Ring Counter is shown below:

Clock	Q_A	Q_B	Q_C	Q_D
0	1	0	0	0
1	0	1	0	0
2	0	0	1	0
3	0	0	0	1
4	1	0	0	0
5	0	1	0	0
6	0	0	1	0
7	0	0	0	1
8	1	0	0	0

Q6. Design 4-bit Johnson counter (Switch Tail Counter) with state table.

Answer: 4-bit Johnson counter is shown below.



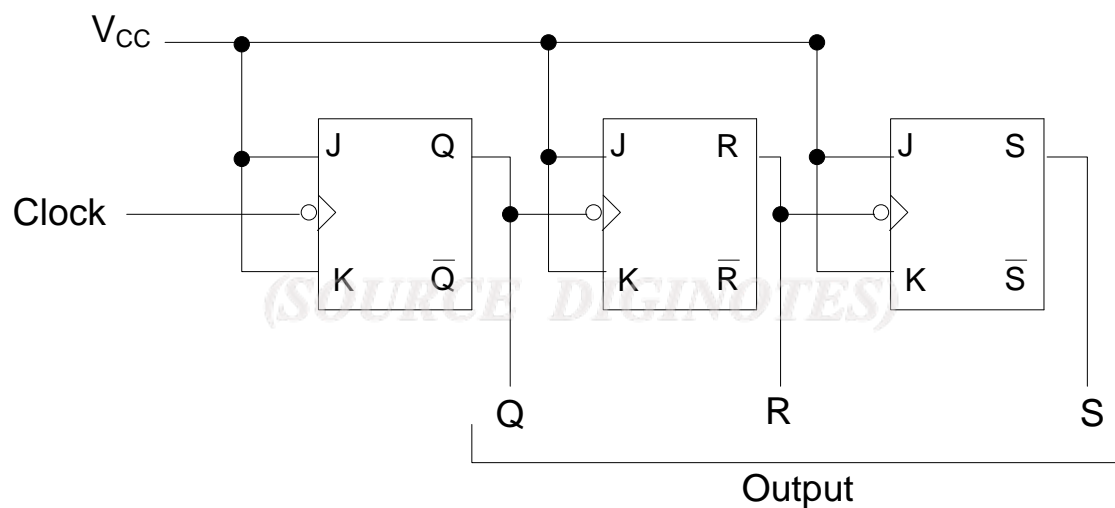
Inverting output of the last flip flop is feedback to the first flip flop.

State table of Johnson Counter is shown below:

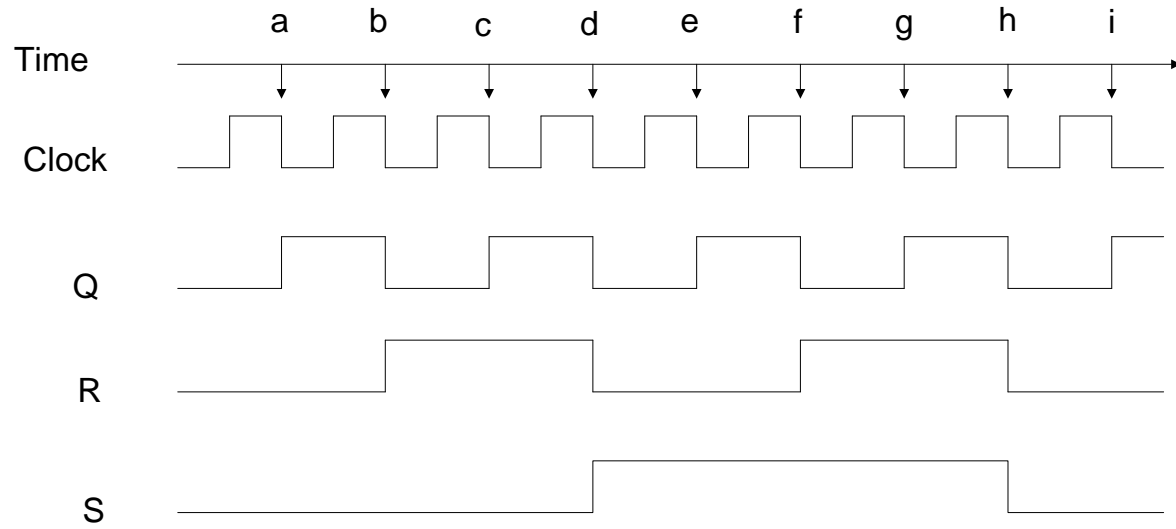
Clock	Serial in= T'	Q	R	S	T
0	1	0	0	0	0
1	1	1	0	0	0
2	1	1	1	0	0
3	1	1	1	1	0
4	0	1	1	1	1
5	0	0	1	1	1
6	0	0	0	1	1
7	0	0	0	0	1
8	1	0	0	0	0
9	1	1	0	0	0

Q7. Explain a 3-bit binary Ripple up counter, give the block diagram, truth table and output waveforms.

Answer: Following is a Ripple Up Counter (or Asynchronous UP Counter).



Waveforms:



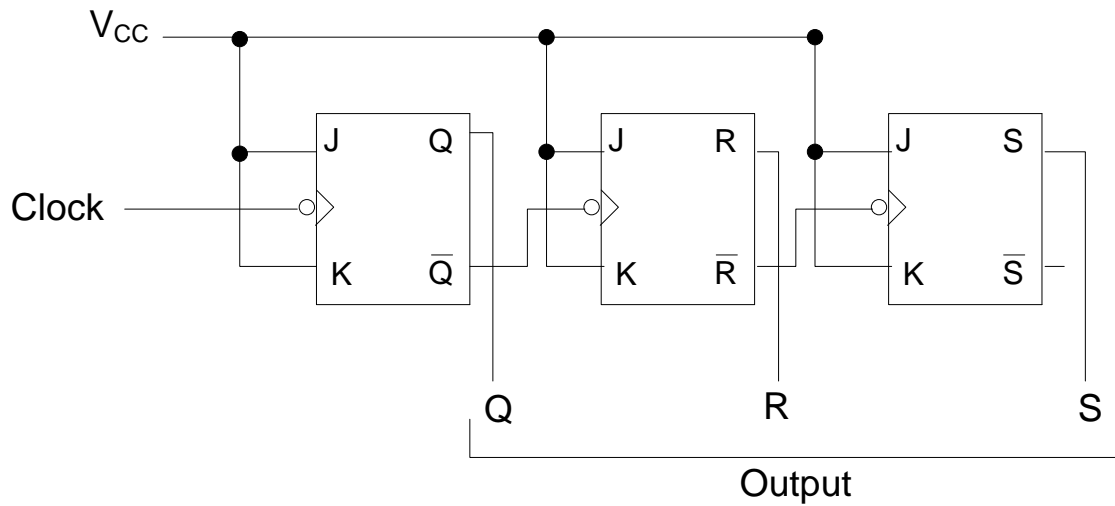
Truth Table:

Negative Edge Triggered Clock	Q	R	S	Count
---	0	0	0	0
a	0	0	1	1
b	0	1	0	2
c	0	1	1	3
d	1	0	0	4
e	1	0	1	5
f	1	1	0	6
g	1	1	1	7
h	0	0	0	0
i	0	0	1	1

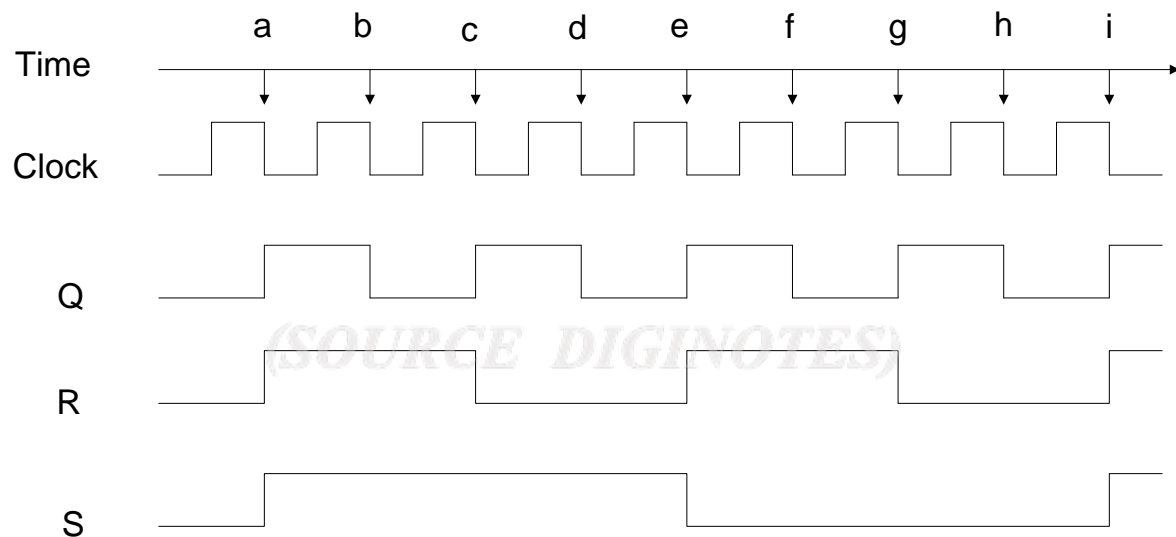
(SOURCE DIGINOTES)

Q8. Explain a 3-bit binary Ripple down counter, give the block diagram, truth table and output waveforms.

Answer: Following is a Ripple Down Counter (or Asynchronous Down Counter).



Waveforms:

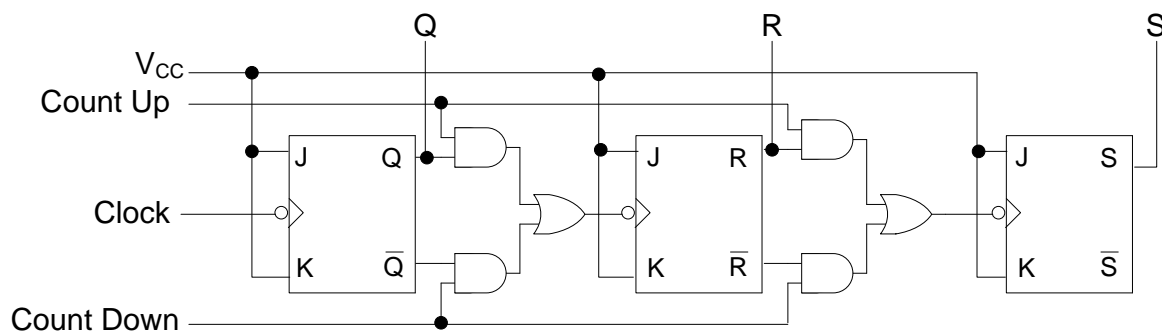


Truth Table:

Negative Edge Triggered Clock	Q	R	S	Count
---	0	0	0	0
a	1	1	1	7
b	1	1	0	6
c	1	0	1	5
d	1	0	0	4
e	0	1	1	3
f	0	1	0	2
g	0	0	1	1
h	0	0	0	0
i	1	1	1	7

Q9. Explain a 3-bit Ripple Up Down Counter.

Answer: Following is a Ripple Up Down Counter.



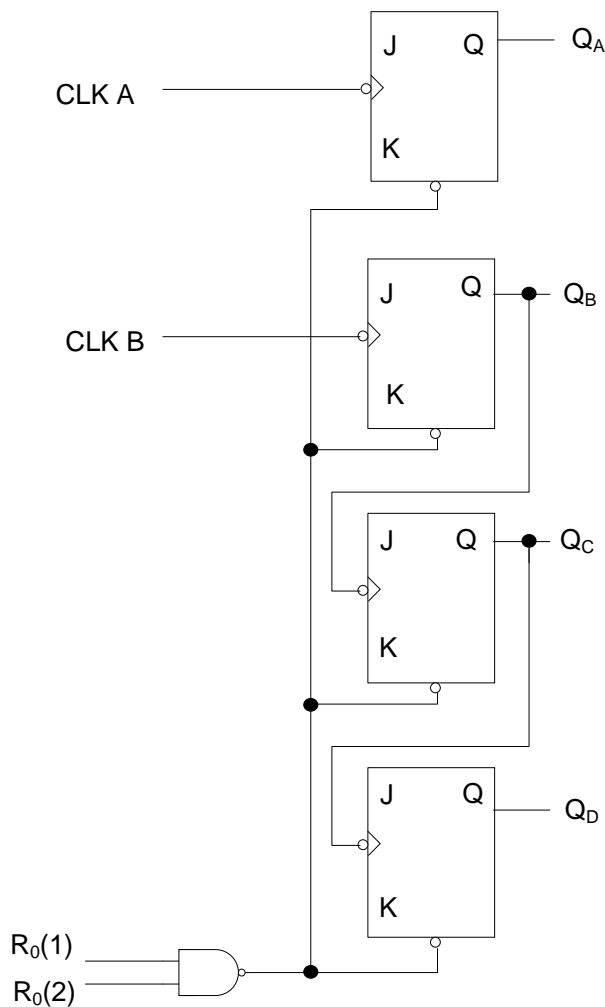
Ripple Up Down Counter is combination Ripple Up Counter and Ripple Down Counter.

If Count Up control is high and Count Down control is low, the above counter will be Up Counter.

If Count Up control is low and Count Down control is high, the above counter will be Down Counter.

Q10. Explain the 7493 IC.

Answer: Following is the 7493 IC



7493 IC is a 4 bit binary counter that can be used in either mod-8 or mod-16.

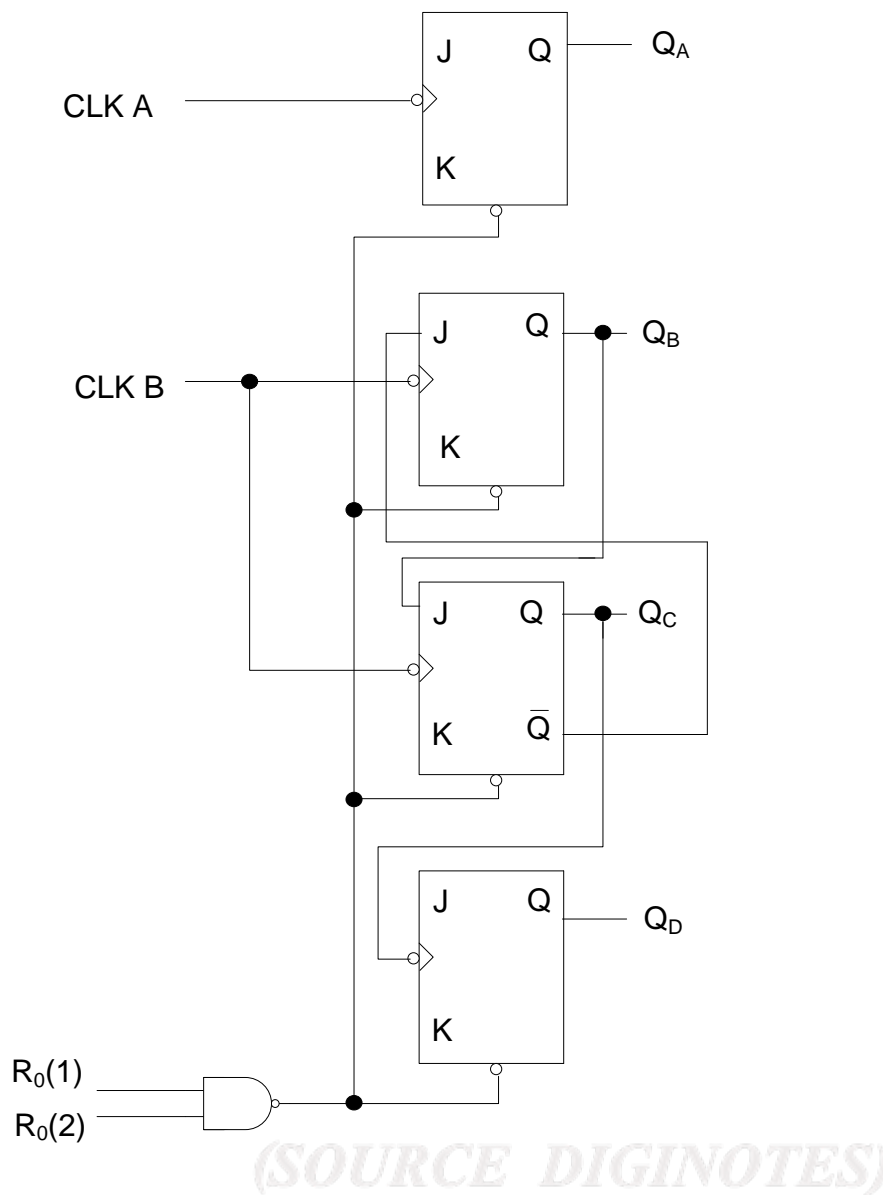
If the clock is applied at CLK B, the counter will be mod-8 counter and the output appear at Q_B , Q_C and Q_D .

If the clock is applied at CLK A and Q_A is connected to CLK B, the counter will be mod-16 Counter and the output appear at Q_A , Q_B , Q_C and Q_D .

$R_0(1)$ and $R_0(2)$ are used to reset all flip flops simultaneously.

Q11. Explain IC 7492.

Answer: Following is the logical diagram for IC 7492



IC 7492 can be used as divide by 12 counter or as divide by 6 counter.

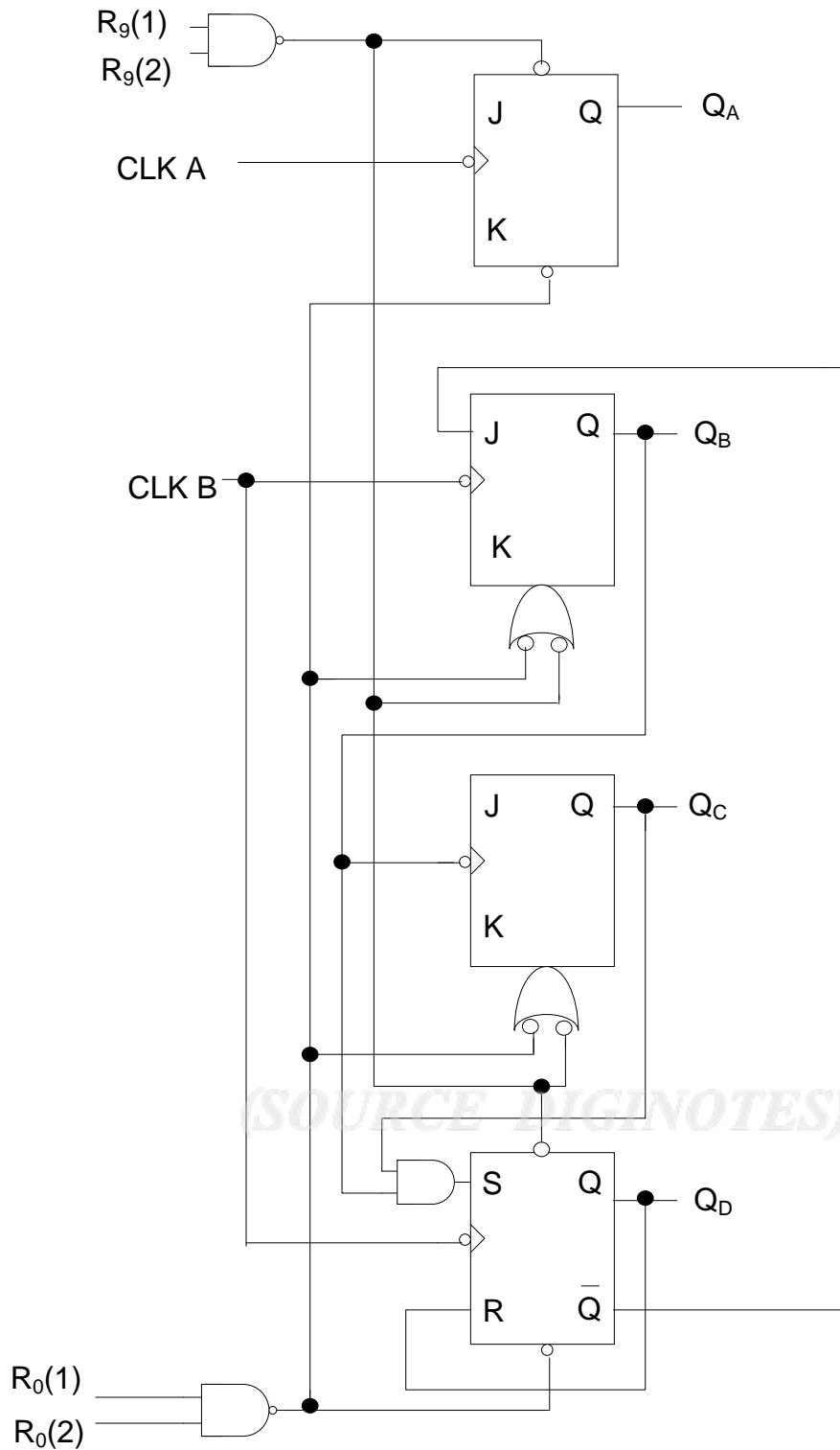
If the clock is applied at input B and the outputs are taken at Q_B , Q_C and Q_D , then the counter is divide by 6 counter.

If the clock is applied at input A, Q_A is connected to input CLK B and the outputs are taken at Q_A , Q_B , Q_C and Q_D , then the counter is divide by 12 counter.

$R_0(1)$ and $R_0(2)$ are used to reset all flip flops simultaneously.

Q12. Explain IC 7490.

Answer: Following is the logical diagram for IC 7490

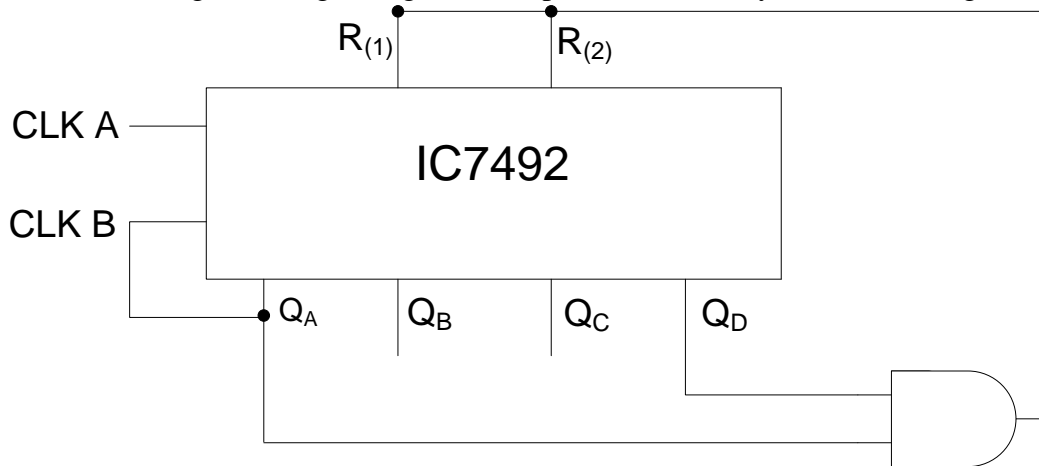


IC 7490 is a decade counter. If the system clock is applied at CLK A and Q_A is connected to CLK B, then the counter is a decade counter.

$R_0(1)$ and $R_0(2)$ are used to reset all flip flops simultaneously.

Q13. Design divide by 9 counter using IC 7492.

Answer: Following is the logic diagram to implement divide by 9 counter using IC 7492.



IC 7492 is a mod-12 counter when Q_A is connected to CLK B.

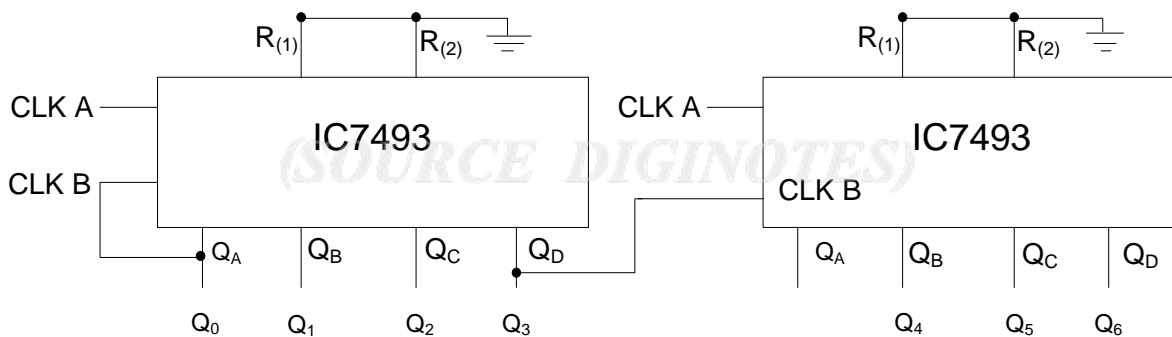
In the above diagram, when $Q_D Q_C Q_B Q_A = 1001$, output of the AND gate is high and as a result, the counter reset as $Q_D Q_C Q_B Q_A = 0000$. Thus, the above circuit is a divide by 9 counter.

Q14. Design a divide by 128 counter using 7493 ICs.

Answer: Following is the logic diagram divide by 128 counter.

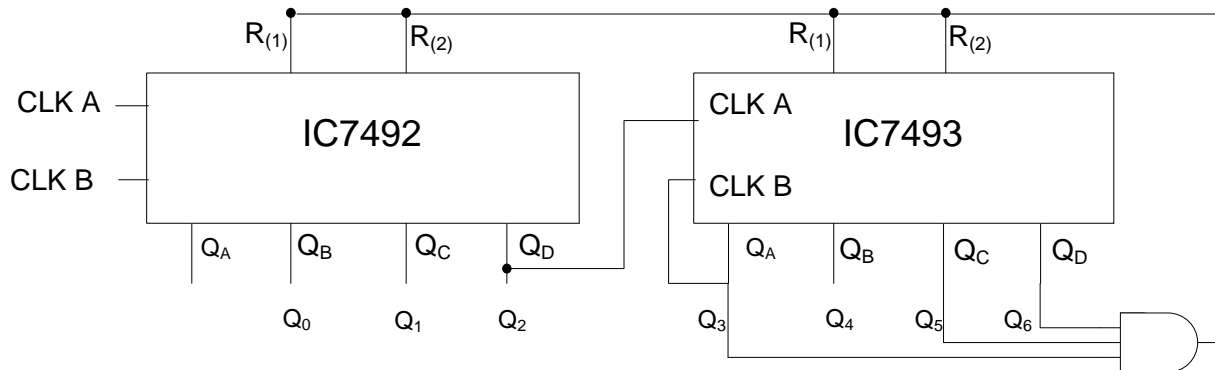
$128 = 16 \times 8$. First IC 7493 is used as divide by 16. The CLK B is input from Q_A and the external clock is applied at CLK A.

Second IC 7493 is used as divide by 8. CLK A is unused. CLK B is input from Q_D of the first IC.



Q15. Design a divide by 78 counter using 7493 and 7492 counter ICs.

Answer: Following is the logic diagram divide by 78 counter.



$$78 = 6 \times 13.$$

IC 7492 is used as divide by 6 counter. CLK A is unused and the external clock is applied to the CLK B.

IC 7493 is to be used as divide by 13 counter. To make this as divide by 13 counter, Q_D of IC 7492 is connected to the CLK A of IC 7493. Q_A , Q_C and Q_D are connected as input to a AND gate and output of the AND gate is connected reset pins.

Q16. Design mod-3 synchronous counter.

Answer:

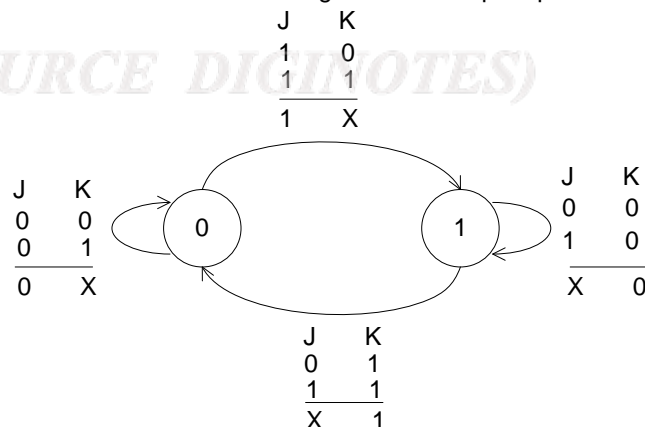
Truth Table

Clock	Counter Output	
	Q_2	Q_1
0	0	0
1	0	1
2	1	0
3	0	0

State Transition Diagram for JK Flip Flop

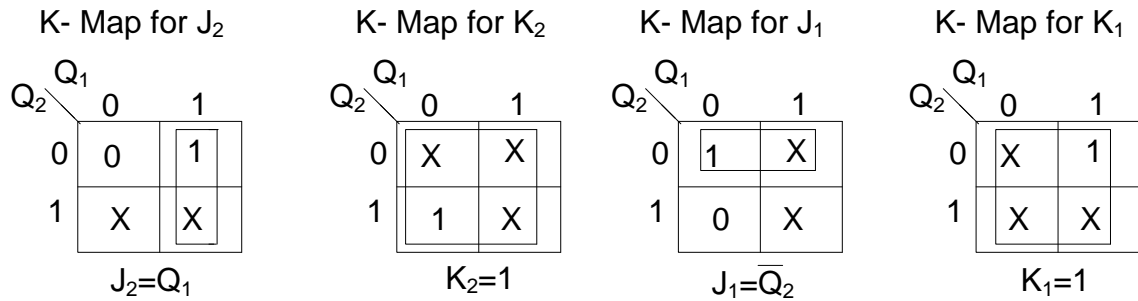
Truth Table for JK Flip Flop

J	K	Q_{n+1}	Action
0	0	Q_n	No Change
0	1	0	Reset
1	0	1	Set
1	1	\bar{Q}_n	Toggle

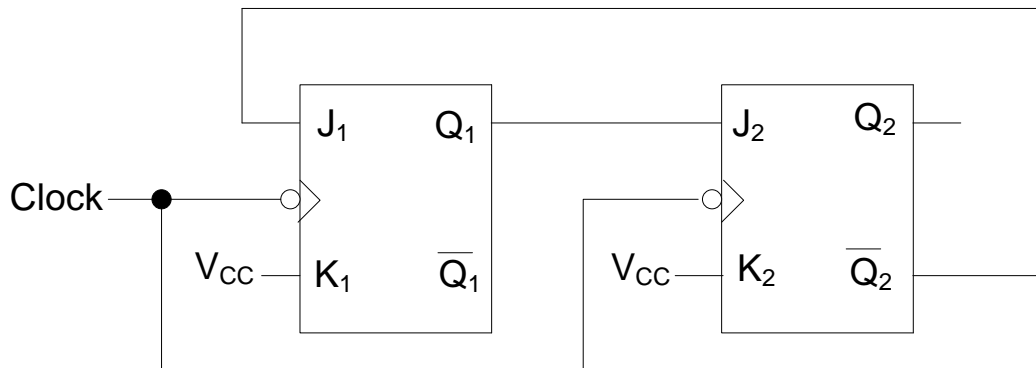


State table for the design of Modulo-3 synchronous counter

Present State		Next State		J_2	K_2	J_1	K_1
Q_2	Q_1	Q_2+	Q_1+				
0	0	0	1	0	X	1	X
0	1	1	0	1	X	X	1
1	0	0	0	X	1	0	X



Logic diagram:



Q17. Design mod-5 synchronous counter.

Answer:

Truth Table:

Clock	Counter Output		
	Q_3	Q_2	Q_1
0	0	0	0
1	0	0	1
2	0	1	0
3	0	1	1
4	1	0	0
5	0	0	0

Truth Table for JK Flip Flop

J	K	Q_{n+1}	Action
0	0	Q_n	No Change
0	1	0	Reset
1	0	1	Set
1	1	\bar{Q}_n	Toggle

State Transition Diagram for JK Flip Flop

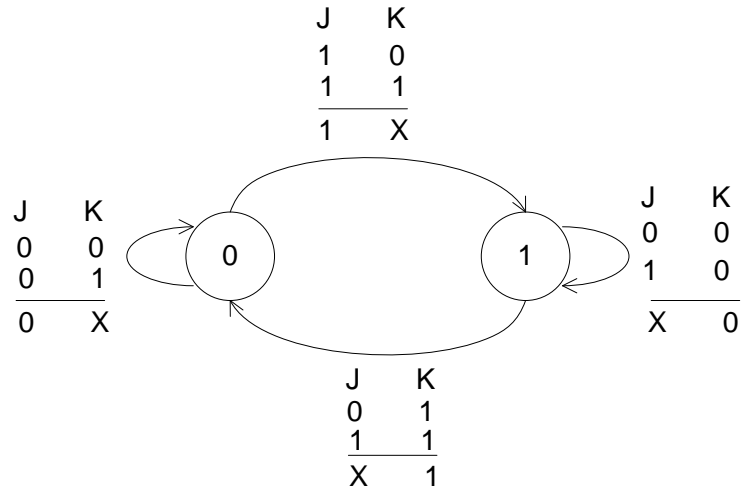


Table for the design of Modulo-5 synchronous counter

Present State			Next State			J_3	K_3	J_2	K_2	J_1	K_1
Q_3	Q_2	Q_1	Q_{3+}	Q_{2+}	Q_{1+}						
0	0	0	0	0	1	0	X	0	X	1	X
0	0	1	0	1	0	0	X	1	X	X	1
0	1	0	0	1	1	0	X	X	0	1	X
0	1	1	1	0	0	1	X	X	1	X	1
1	0	0	0	0	0	X	1	0	X	0	X

K-map for J_3 :

Q_3	$Q_2 Q_1$				
	00	01	11	10	
0	0	0	1	0	$J_3 = Q_2 Q_1$
1	X	X	X	X	

K-map for K_3 :

$Q_3 \backslash Q_2 Q_1$		00	01	11	10	
0		X	X	X	X	$K_3=1$
1		1	X	X	X	

K-map for J_2 :

$Q_3 \backslash Q_2 Q_1$		00	01	11	10	
0		0	1	X	X	$J_2=Q_1$
1		0	X	X	X	

K-map for K_2

$Q_3 \backslash Q_2 Q_1$		00	01	11	10	
0		X	X	1	0	$K_2=Q_1$
1		X	X	X	X	

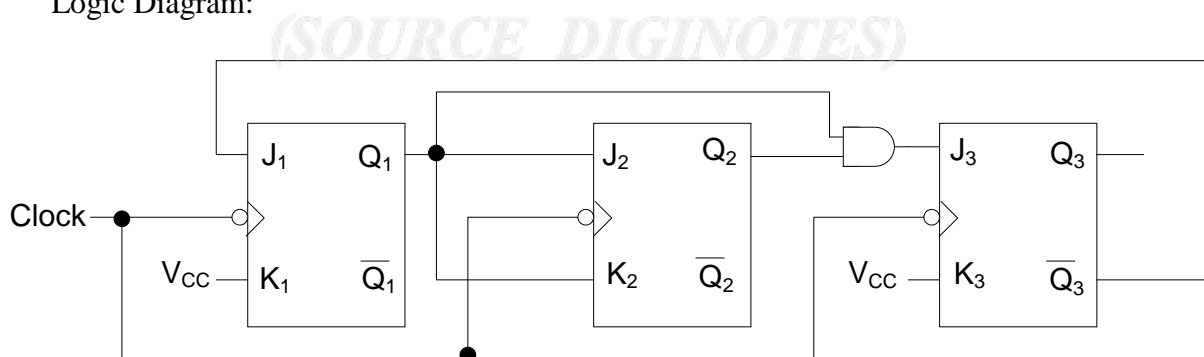
K-map for J_1 :

$Q_3 \backslash Q_2 Q_1$		00	01	11	10	
		0	1	1	0	
0	1	X	X	1		$J_1 = \bar{Q}_3$
1	0	X	X	X		

K-map for K_1 :

$Q_3 \backslash Q_2 Q_1$		00	01	11	10	
		0	1	1	0	
0	X	1	1	X		$K_1 = 1$
1	1	X	X	X		

Logic Diagram:



Q18. Design synchronous mod-6 counter.

Answer:

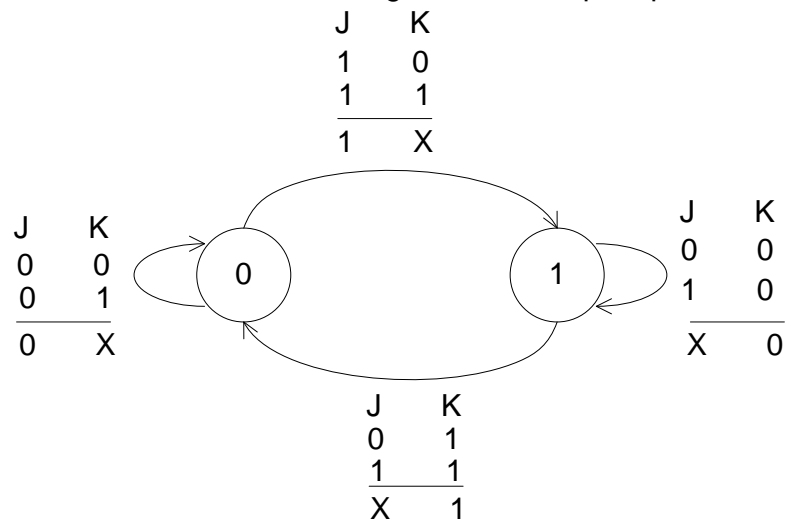
Truth Table:

Clock	Counter Output		
	Q ₃	Q ₂	Q ₁
0	0	0	0
1	0	0	1
2	0	1	0
3	0	1	1
4	1	0	0
5	1	0	1
6	0	0	0

State Transition Diagram for JK Flip Flop

Truth Table for JK Flip Flop

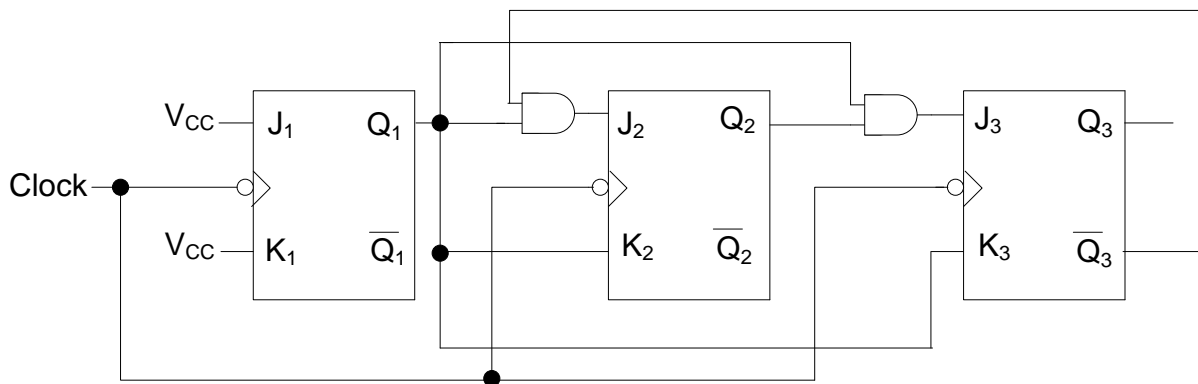
J	K	Q _{n+1}	Action
0	0	Q _n	No Change
0	1	0	Reset
1	0	1	Set
1	1	\overline{Q}_n	Toggle



State Table for the design of Modulo-5 synchronous counter

Present State			Next State			J ₃	K ₃	J ₂	K ₂	J ₁	K ₁
Q ₃	Q ₂	Q ₁	Q ₃₊	Q ₂₊	Q ₁₊						
0	0	0	0	0	1	0	X	0	X	1	X
0	0	1	0	1	0	0	X	1	X	X	1
0	1	0	0	1	1	0	X	X	0	1	X
0	1	1	1	0	0	1	X	X	1	X	1
1	0	0	1	0	1	X	0	0	X	1	X
1	0	1	0	0	0	X	1	0	X	X	1

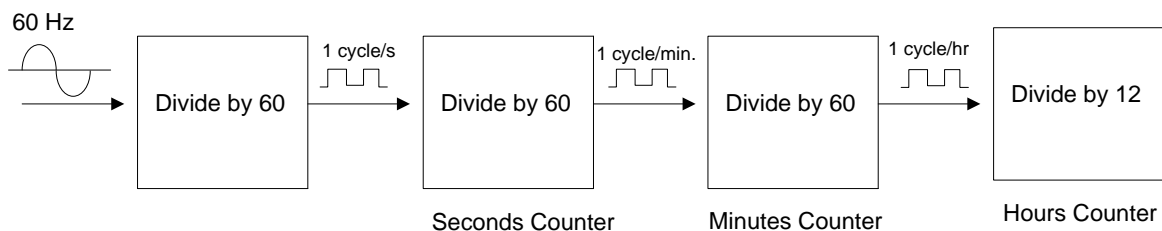
Circuit diagram:



Q19. Explain Digital Clock with block diagram.

Answer: In several countries power supply is 50Hz. There one can use standard variable frequency signal generator to get 60Hz.

Block diagram of a digital clock:



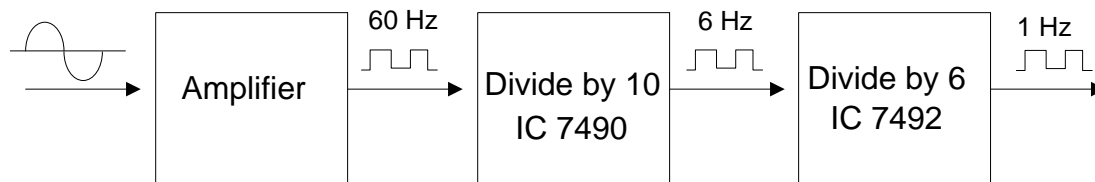
Block diagram shows the functions to be performed. The first divide by 60 counter divides the 60 Hz power signal down to a 1 Hz square wave. This 1 Hz square wave is the input to the second counter.

The second divide by 60 counter changes its state once each second and has 60 discrete states. It can be decoded to provide signal to display second. This counter produces output square wave of 1 cycle per minute and this is the input to the third counter.

The third divide by 60 counter changes its state once each minute and has 60 discrete states. It can be decoded to provide signal to display minute. This counter produces output square wave of 1 cycle per hour and this is input to the fourth counter.

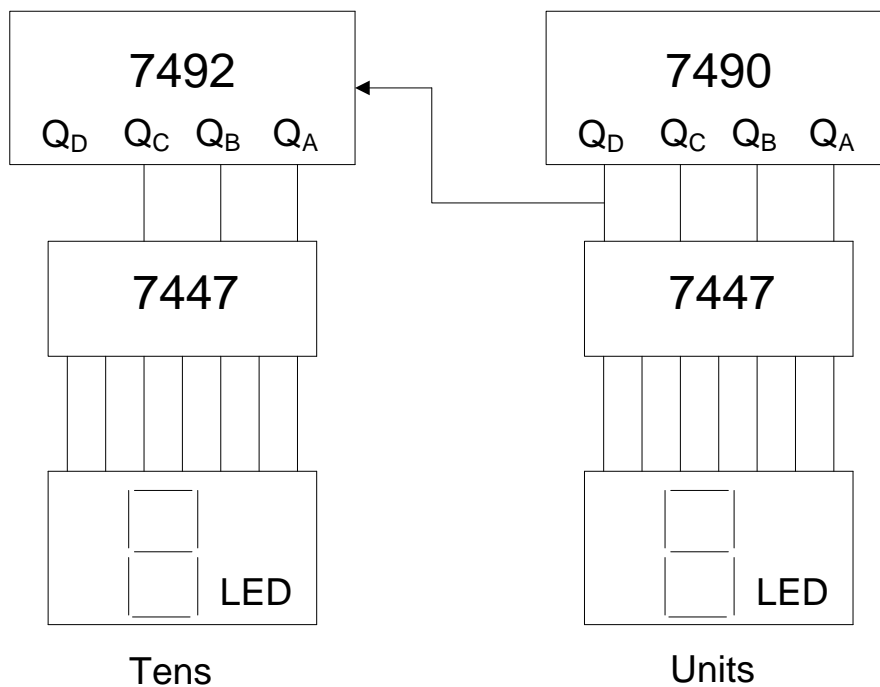
The last counter changes its state once each hour and has 12 discrete states. It can be decoded to provide signal to display hour. The last counter reset at every 12 hours.

Divide by 60 counter can be implemented by cascading divide by 10 counter (IC 7490) and divide by 6 counter (IC 7492). This is in the block diagram below.



.Display of Second, Minute and Hour can be implemented by the IC 7447 and 7 segment display.

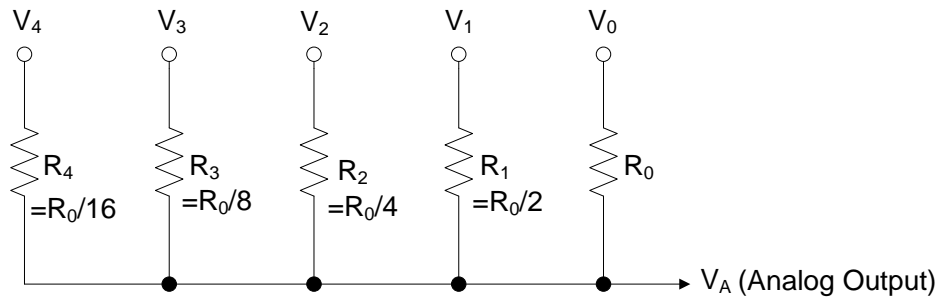
This is shown below in the following diagram.



(SOURCE DIGINOTES)

Q1. Explain 5-bit resistive divider with diagram.

Answer: 5-bit resistive divider is shown below:



A resistive divider can be build to change a digital voltage to an equivalent analog voltage.

The following criterion can be applied to resistive divider.

- There must be one input resistor for each digital bit.
- Beginning with the LSB, each following resistor value is one half of the previous resistor.
- The LSB has weight of $\frac{1}{2^n - 1}$, where n is the number of input bits.
- The change in output voltage due to a change in the LSB is equal to $\frac{V}{2^n - 1}$ where V is the digital input voltage.
- The output voltage can be obtained for any digital input signal by following equation.

$$V_A = \frac{V_0 2^0 + V_1 2^1 + V_2 2^2 + V_3 2^3 + \dots + V_{n-1} 2^{n-1}}{2^n - 1}$$
, where $V_0, V_1, V_2, \dots, V_{n-1}$ are the digital voltage (0 or V) and n is the number of input bits.

Q2. For a 5-bit resistive divider, determine the following (a) the weight assigned to the LSB

(b) the weight assigned to the second and the third LSB (c) the change in output voltage

due to a change in the LSB, the second LSB, and the third LSB (d) the output voltage for a digital input 10101. Assume 0= 0 V and 1=+10 V.

Answer:

(a) The LSB weight is $\frac{1}{2^5 - 1} = \frac{1}{31}$

(b) The second LSB weight is $\frac{2}{31}$ and third LSB weight is $\frac{4}{31}$

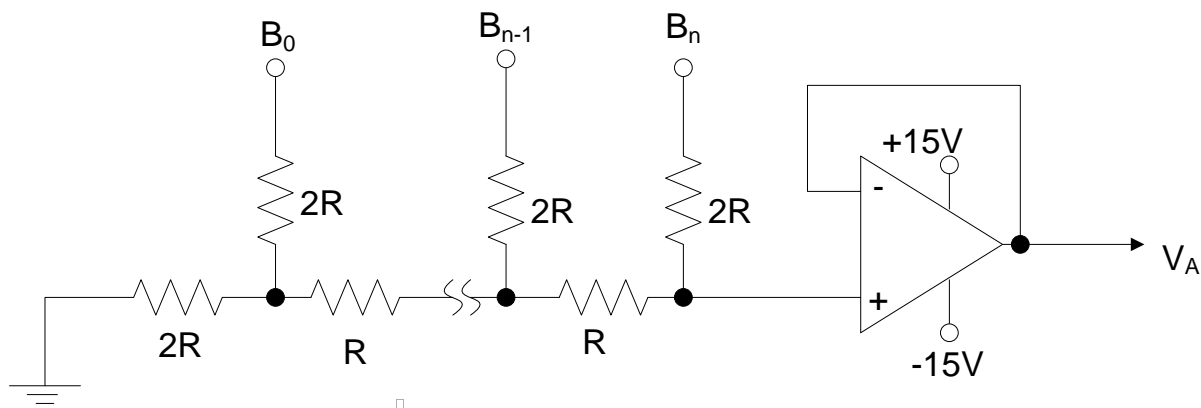
(c) The LSB causes a change in the output voltage of $\frac{10}{31}$ V. The second LSB causes an output voltage change of $\frac{20}{31}$ V and third LSB causes an output voltage change of $\frac{40}{31}$ V.

(d) The output voltage for a digital input of 10101 is

$$V_A = \frac{10 \times 2^0 + 0 \times 2^1 + 10 \times 2^2 + 0 \times 2^3 + 10 \times 2^4}{2^5 - 1} = \frac{10(1 + 4 + 16)}{31} = \frac{210}{31} = 6.77 \text{ V}$$

Q3. Explain binary ladder with diagram.

Answer: Binary ladder is shown below.



B_n, B_{n-1}, \dots, B_0 are the digital inputs, whose values are either 0 (0 Volt) or 1 (1 Volt).

B_n is MSB and B_0 is the LSB.

$$V_A = V \left(B_n \times \frac{1}{2} + B_{n-1} \times \frac{1}{4} + \dots + B_0 \times \frac{1}{2^n} \right)$$

Q4. Find the output voltage from a 5-bit ladder that has a digital input of 11010. Assume that

0=0 V and 1=+10 V.

Answer:

$$V_A = 10 \left(1 \times \frac{1}{2} + 1 \times \frac{1}{4} + 0 \times \frac{1}{8} + 1 \times \frac{1}{16} + 0 \times \frac{1}{32} \right) = 10 \left(\frac{1}{2} + \frac{1}{4} + \frac{1}{16} \right) = 8.125 \text{ V}$$

Q5. Explain the terms accuracy and resolution for D/A converter.

Answer:

Accuracy is a measure of how close the actual output voltage is to the theoretical output value.

For example, suppose the theoretical output voltage for a particular input is +10 V.

For accuracy of 1 percent, the actual output voltage must lie between +9.9 V and +10.1 V.

Resolution defines the smallest increment in voltage that can be recognized. Resolution is a function of number of bits in the digital input signal. In a 4-bit ladder system, the LSB weight is $1/16$. This means that the smallest increment in output voltage is $1/16$ of the input voltage.

If the input voltage is +16 V, then the output voltage changes in steps of 1 V. This converter cannot resolve voltages smaller than 1 V. This converter is not capable of distinguishing voltages finer than 1 V which is the resolution of the converter.

Q6. What is the resolution of a 9-bit D/A converter which uses a ladder network? What is the resolution expressed as a percentage? If the full scale output voltage of this converter is +5 V, What is the resolution in volts?

Answer:

In a 9-bit system, the LSB has a weight of $\frac{1}{2^9} = \frac{1}{512}$.

Hence, the resolution of the converter expressed in percentage = $\frac{1}{512} \times 100\% \cong 0.2\%$

The resolution in volt = $\frac{1}{512} \times 5 \cong 10\text{mV}$

Q7. How many bits are required at the input of a converter if it is necessary to resolve voltage to 5 mV and the ladder has +10 V full scale?

Answer:

Resolution = $\frac{1}{2^n} \times \text{full scale voltage}$ where n is the number of bits

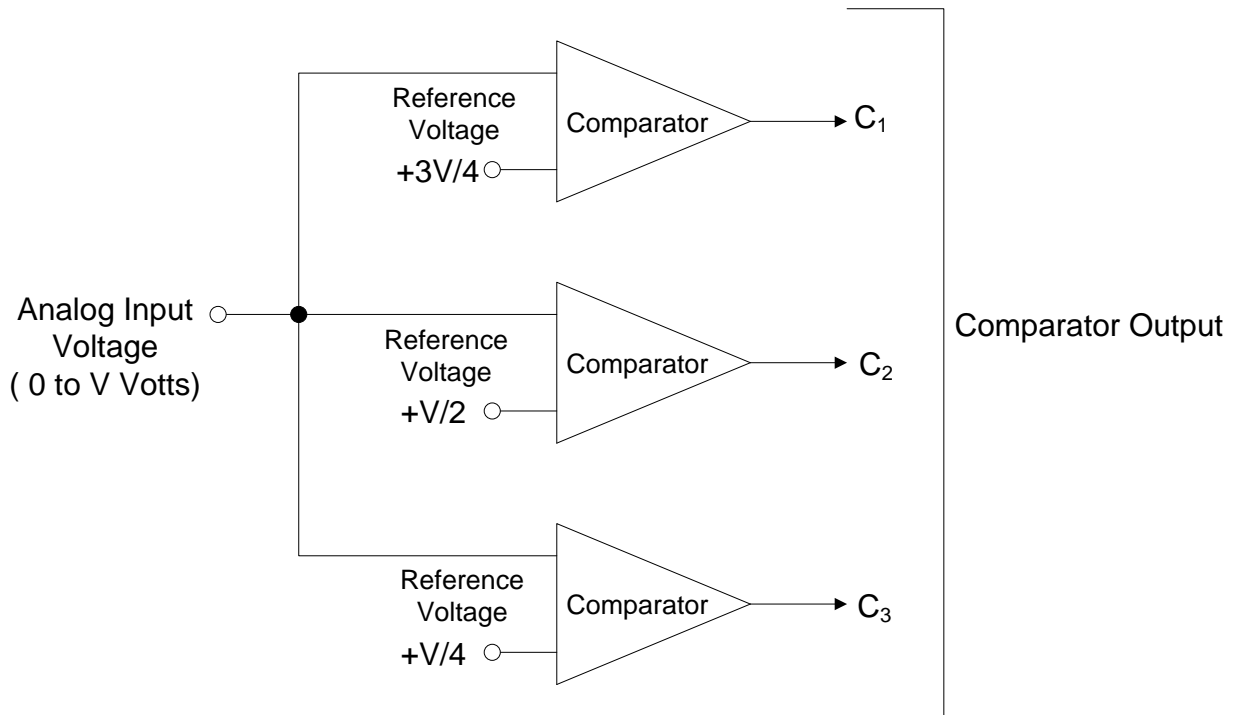
$$\Rightarrow 5 \times 10^{-3} = \frac{1}{2^n} \times 10$$

$$\Rightarrow 2^n = 2000$$

$$\Rightarrow n \cong 11$$

Q8. Explain simultaneous A/D converter with diagram.

Answer: Following is the logical diagram for 2-bit simultaneous A/D converter.



Following table shows the comparator output for input voltage ranges

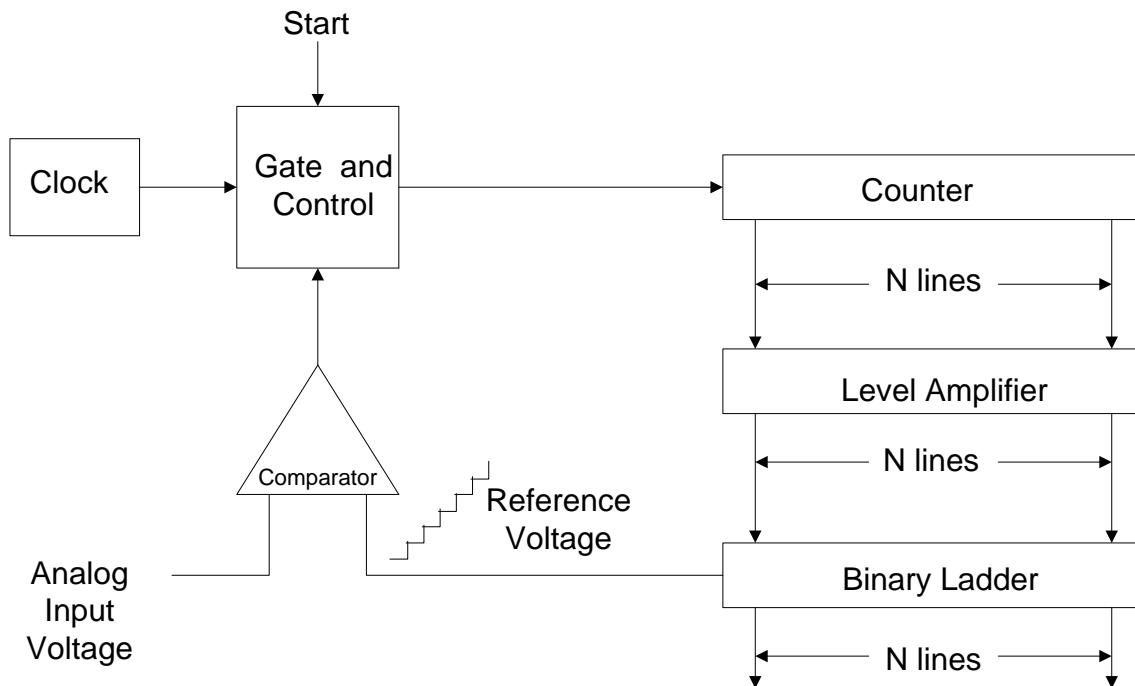
Input Voltage	Comparator Output		
	C ₃	C ₂	C ₁
0 to +V/4	Low	Low	Low
+V/4 to +V/2	Low	Low	High
+V/2 to +3V/4	Low	High	High
+3V/4 to V	High	High	High

The simultaneous method of A/D conversion using three comparators is shown in the above figure. The analog signal to be digitized serves as one of the inputs to each comparator. The second input is a standard reference voltage. The reference voltages used are +V/4, +V/2 and +3V/4. The system is then capable of accepting an analog input voltage between 0 and +V.

If the analog signal exceeds the reference voltage to any comparator, that comparator turns on. Now, if all the comparator are off, the analog input signal must be between 0 and +V/4. If C₁ is high and C₂ and C₃ are low, the input must be between +V/4 and +V/2. If C₁ and C₂ are high and C₃ is low, the input must be between +V/2 and +3V/4. If all the comparator outputs are high, the input signal must be between +3V/4 and +V.

Q9. Explain counter type A/D converter with diagram.

Answer: Following is the counter type A/D converter



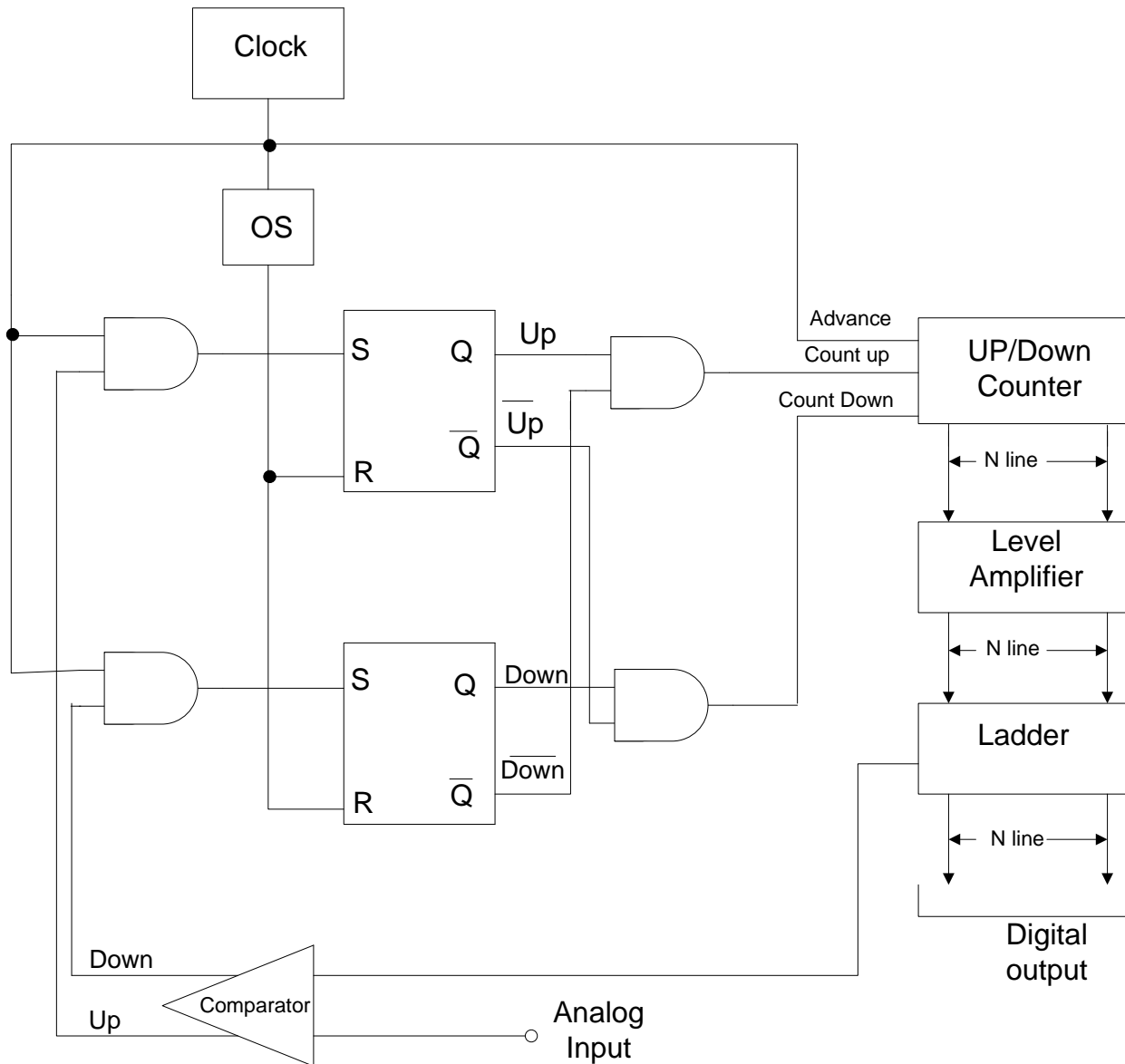
This type of A/D converter consists of binary counter. The digital output signal of this counter is connected to a standard binary ladder D/A converter. If a clock is applied to the input of the counter, the output of the binary ladder D/A converter is the staircase waveform. This waveform is the reference voltage signal for the comparator.

First, the counter is reset to all 0s. Then, when a convert signal appears on the START line, the gate opens and the clock pulses are allowed to pass through to the input of the counter.

The counter advances through a normal binary count sequence, and the staircase waveform is generated at the output of the ladder. This waveform is applied to one side of the comparator and analog input voltage is applied to the other side. When the reference voltage equals (or exceeds) the input analog voltage, the gate is closed, the counter stops and the conversion is complete. The number stored in the counter is now the digital equivalent of the analog input voltage.

Q10. Explain continuous A/D converter with diagram.

Answer: Following is the continuous A/D converter:



(SOURCE DIGINOTES)

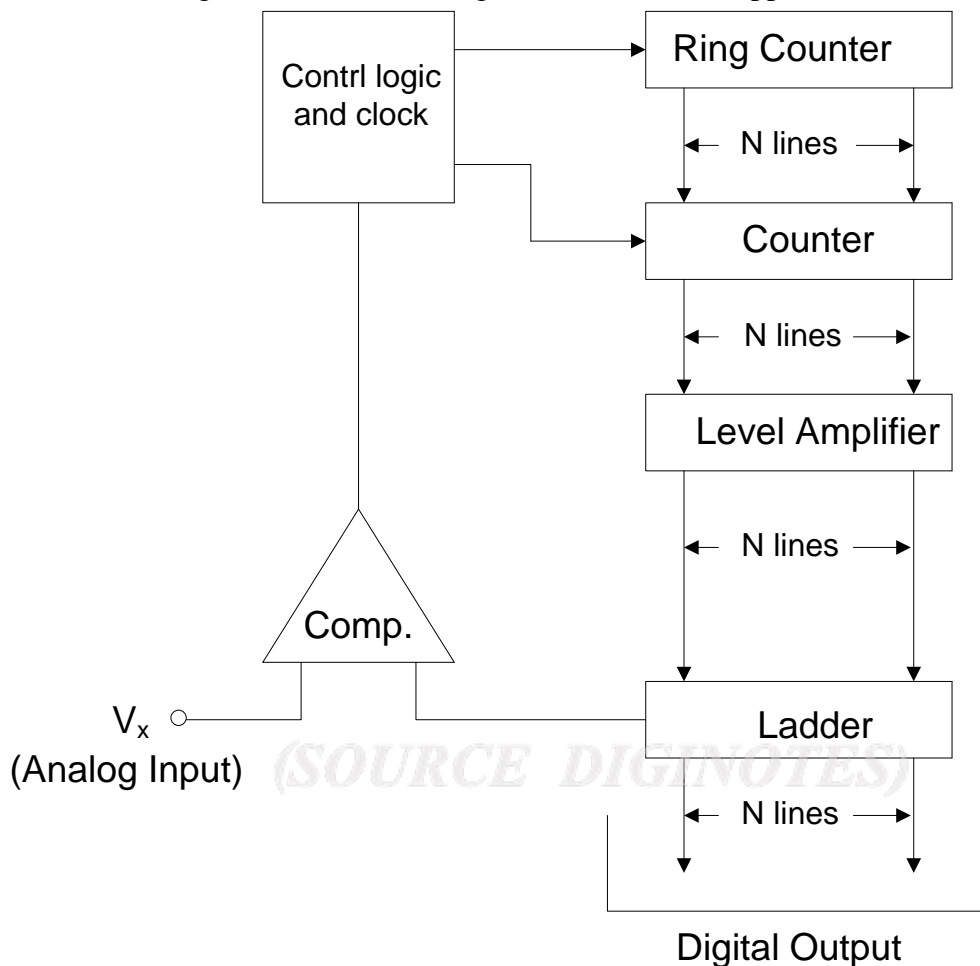
This type of A/D converter consists of binary up/down counter. The digital output signal of this counter is connected to a standard binary ladder to form a D/A converter. Output of the ladder is fed into a comparator which has two outputs. When the input analog voltage is more positive than the ladder output, Up output of the comparator is high. When the input analog voltage is more negative than the ladder output, the down output is high.

If the Up output of the comparator is high, the AND gate at the input of the Up flipflop opens, and the first time the clock goes positive, the flipflop is set. At the moment the down flipflop is reset, the AND gate which controls the count-up line of the counter will be true and the counter advances one count. As long as the Up line out of the comparator is high, the converter continues to operate and advances its count.

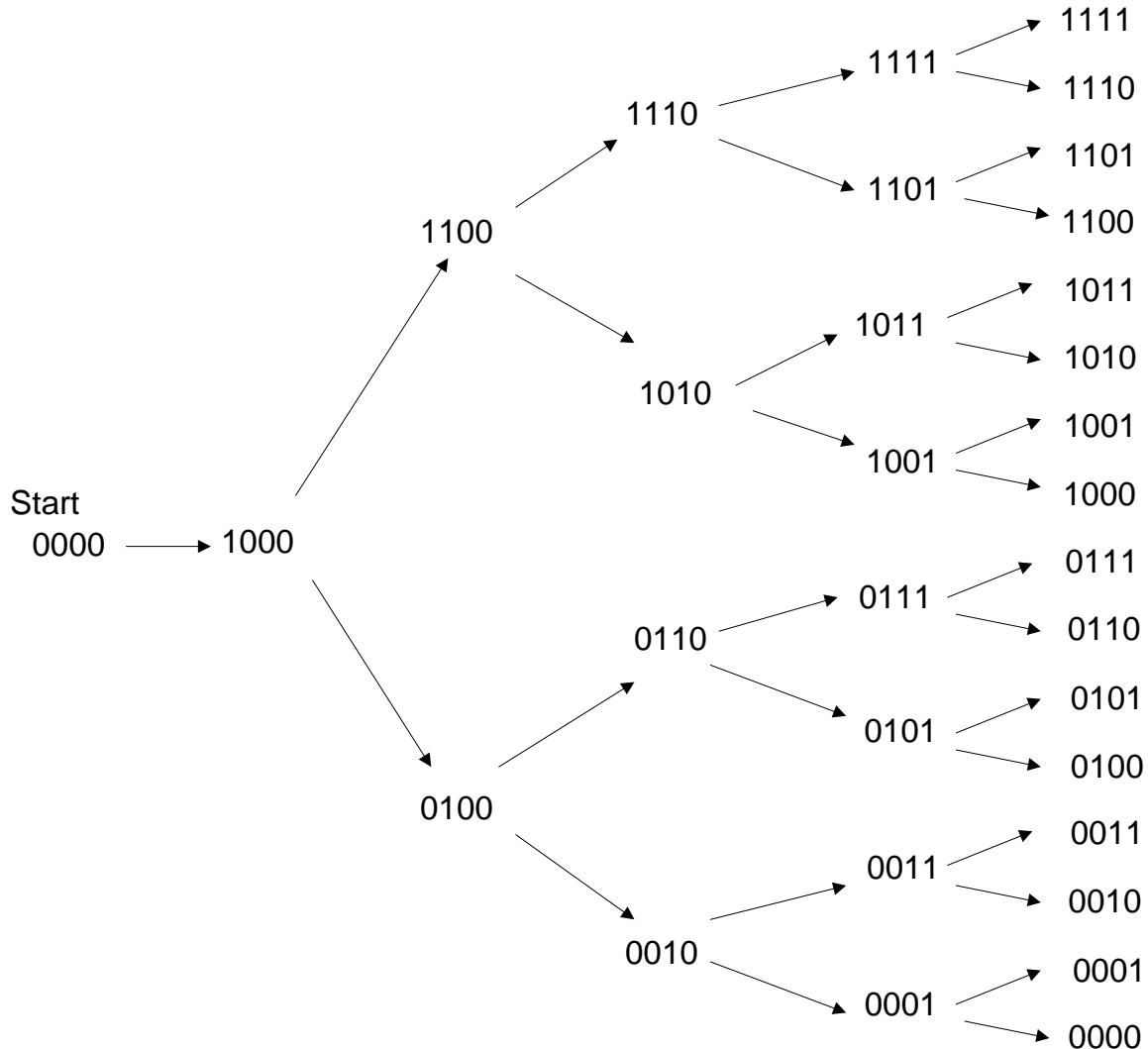
At the point where the ladder voltage becomes more positive than the input voltage, the Up line of the comparator goes low and the Down line goes high. The converter then goes through a count-down conversion cycle.

Q11. Explain successive approximation converter with diagram.

Answer: Following shows the block diagram for successive approximation converter.



Following shows the operation performed in successive approximation converter



Successive approximation converter consists of counter which is first reset to all 0s. The MSB of the counter is then set. The MSB is then left in or taken out (by resetting the MSB flipflop) depending on the output of the comparator. Then the second MSB is set in, and comparator is made to determine whether to reset the second MSB flipflop. The process is repeated down to LSB and at this time desired number is in the counter. The converter operates by successive dividing the voltage ranges in half. The successive approximation method is the process of approximating the analog voltage by trying 1 bit at a time beginning with MSB. The operation is shown in the above diagram.